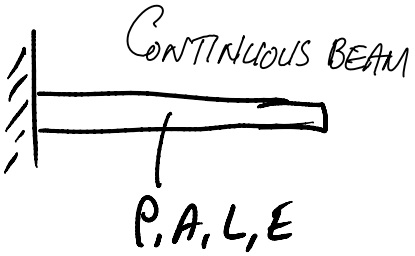
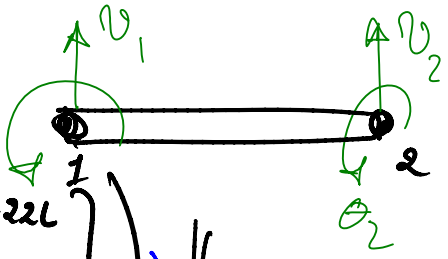
CLOSED-FORM SOLUTION EXAMPLE:



CHARACTERISTIC EQ'N:

$$\left([k] - \omega^2 [m] \right) \{\delta\} = \{0\}$$

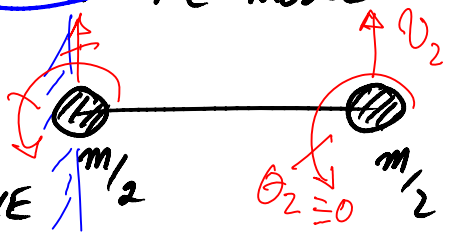
ONE ELEM. APPROX.



APPLY SINGLE "BEAM" ELEMENT STIFFNESS & MASS MATRIX:

$$\left(EI \begin{bmatrix} 12 & -6L \\ -6L & 4L^2 \end{bmatrix} - \omega^2 \left(\frac{\rho AL}{420} \right) \begin{bmatrix} 156 & -22L \\ -22L & 4L^2 \end{bmatrix} \right) \begin{Bmatrix} v_2 \\ \theta_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

F.E. MODEL



FOR NON-TRIVIAL SOLUTION, DETERMINANT OF EXPRESSION INSIDE

$$\left([\,] - \omega^2 [\,] \right) = 0$$

THUS:

$$\begin{vmatrix} 12 - 156a & -6L + 22La \\ -6L + 22La & 4L^2 - 4L^2 a \end{vmatrix} = 0$$

$$a = \frac{\omega^2 \rho A L^4}{420 EI}$$

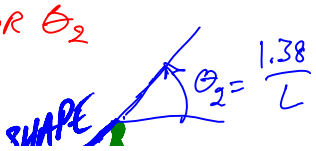
SOLVE TO OBTAIN:

$$\omega_1 = 3.533 \left(\frac{EI}{\rho AL^4} \right)^{1/2} \qquad \omega_2 = 34.81 \left(\frac{EI}{\rho AL^4} \right)^{1/2}$$

MODE 1 SET $v_2 = 1$

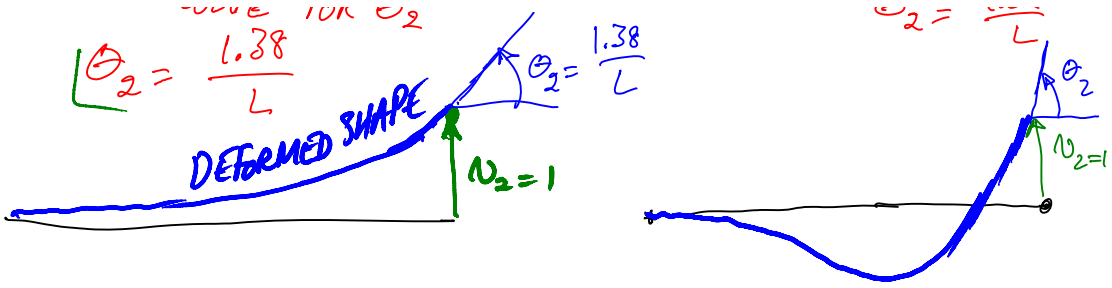
SOLVE FOR θ_2

$$\theta_2 = \frac{1.38}{L}$$



MODE 2 SET $v_2 = 1$

$$\theta_2 = \frac{2.62}{L}$$



LUMPED MASS APPROXIMATION:

$$\omega_1 = 2.449 \left(\frac{EI}{\rho A L^4}\right)^{1/2} \qquad \omega_2 = 0$$

ANALYTICAL (CLOSED FORM) SOLUTION

$$\omega_1 = 3.516 \left(\frac{EI}{\rho A L^4}\right)^{1/2}$$

$$\omega_2 = 22.03 \left(\frac{EI}{\rho A L^4}\right)^{1/2}$$