DYNAMIC (VIBRATION) ANALYSIS:

ADDRESS SOLUTION OF GENERAL EQUATION OF MOTION:

$$\underbrace{m}_{\text{MASS}}\ddot{x} + \underbrace{c}_{\text{DAMPING COEFF.}}\dot{x} + \underbrace{k}_{\text{STIFFNESS}}x = \underbrace{f(t)}_{\text{FORCING FUNCTION}}$$

GENERALLY TWO TYPES OF PROBLEMS:

1) STRUCTURAL DYNAMICS

*- MODAL ANALYSIS (EXTRACT MODES AND NATURAL FREQ'S)

- FREQ. RESPONSE ANALYSIS (APPLY FORCING FUNCTION AND COMPUTE DYNAMIC STRESS & DISPLACEMENT)

LOWER NAT'L FREQ'S

2) WAVE PROPAGATION: SUDDEN LOAD APPLICATION

IMPACT OR EXPLOSIVE (SUDDEN) LOAD.

HIGH NAT'L FREQ'S.

MODE & NAT'L FREQ. EXTRACTION:

SOLVE SIMPLE UNDAMPED FREE VIBRATION.

$$m\ddot{x} + kx = 0$$

MATRIX FORM: $[m]\{\ddot{x}\} + [k]\{x\} = \{0\}$

$x \rightarrow$ ASSUMED DISP. FIELD

$$\{x\} = [N]\{d\} \Rightarrow \{\ddot{x}\} = [N]\{\ddot{d}\}$$

$$\{x\} = [N]\{d\} \Rightarrow \{\ddot{x}\} = [N]\{\ddot{d}\}$$

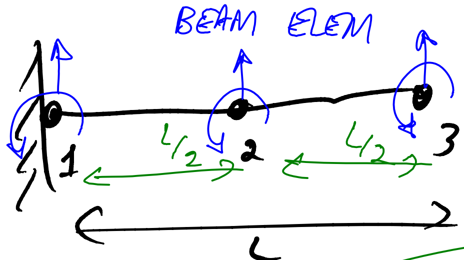
$\{\ddot{x}\}$: ASSUMED ACCEL. FIELD $\{\ddot{d}\}$: NODAL ACCEL. COMPONENTS

MASS MATRIX

$[m]$ TWO METHODS FOR DEVELOPMENT OF MASS MATRIX.

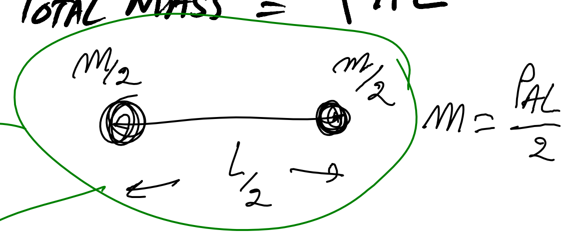
LUMPED MASS APPROXIMATION :

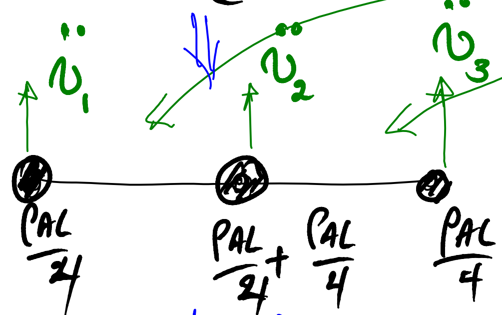
PLACE LUMPED MASSES AT NODAL D.O.F.'S.



ρ, A, L

TOTAL MASS = $\rho A L$





$$[m] = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & & & & & & \\ 2 & & & & & & \\ 3 & & & & & & \\ 4 & & & & & & \\ 5 & & & & & & \\ 6 & & & & & & \end{array} \begin{bmatrix} \frac{\rho A L}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\rho A L}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{\rho A L}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

ROTATIONAL INERTIA TERMS

ONE ASSUMPTION FOR ROTATIONAL INERTIA IS TO BASE IT ON MOMENT REQUIRED TO ROTATE A POINT MASS BY A UNIT ANGULAR ACCELERATION.

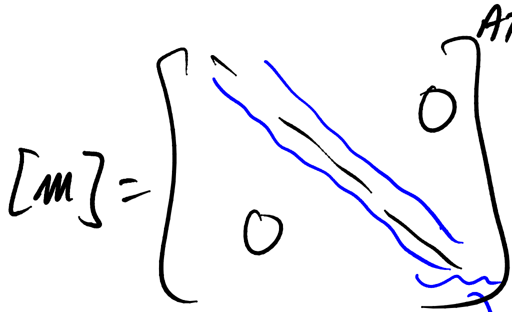
TURNS OUT TO BE $\rho A L^3$ ... BASED ON ELEM LENGTH

TURNS OUT TO BE : $\frac{\cdots}{24}$ BASED ON ELEM. LENGTH of $l/2$

FORMAL MASS LUMPING TECHNIQUES:

- (HRZ - LUMPING) WHICH TAKES DIAGONAL TERMS OF THE CONSISTENT MASS MATRIX, AND SCALES IT TO BE EQUIV. TO TOTAL MASS.

- OPTIMAL LUMPING: PLACE LUMPED MASS TERMS AT GAUSS POINT LOCATION.



TRI-DIAGONAL LUMPING

$$[m] = \int_{Vol.} \rho [N]^T [N] \, dvol.$$

CONSISTENT MASS MATRIX.

Solution of EIGEN-VALUE PROBLEM:

$$[m]\{\ddot{u}\} + [k]\{u\} = \{0\} \quad (1)$$

$$\{\bar{u}\} = \{q\} \sin \omega t \qquad \{\ddot{u}\} = -\{q\}\omega^2 \sin \omega t$$

SUBST. IN (1):

$$\left([k] - \omega^2 [m]\right)\{q\} = \{0\}$$

EIGEN VALUE

EIGEN VECTOR

VALUE VECTOR

$\omega^2 = \lambda$ = EIGEN VALUE

ω -> NAT'L FREQ'S

FOR EACH ω_i THERE EXISTS A $\{\delta\}_i$ MODE SHAPE.

IF $\omega_i = 0 \Rightarrow$ RIGID BODY MOTION.

SOLUTION APPROACHES.

1) DIRECT APPROACH --> SUITABLE FOR SMALL PROBLEMS

2) VECTOR ITERATION -> TAKES ADVANTAGE OF SYMMETRY OF MASS & STIFFNESS MATRICES
   - SUBSPACE ITERATION
   - POWER ITERATION
   - INVERSE POWER ITERATION

3) TRANSFORMATION METHOD - MOST GENERAL COMPUTATIONAL METHOD.
   SIMPLIFY MASS MATRIX BY TRI-DIAGONALIZATION (USING CONSISTENT MASS MATRIX).
   COMPUTATIONALLY INTENSIVE

   LANCZOS METHOD