ME 455/555
Intro. To FE modeling and Analysis

Modeling, Accuracy and Error in Finite Element Analysis

A summary

Grasp of the problem area

FE modeling is more than preparing a mesh and pre/post processing.

Skill in modeling is based on the visualization of physical behavior and relating it to the specific element behavior.

Acquire broad knowledge base.

Practice and conduct critical evaluation of results.

General Guidelines

Know and remember the assumptions and underlying restrictions of analysis tools.

Always start simple and be prepared to revise your model, frequently.

The single most important underlying question in FE modeling and analysis:

"Which element should I use for a given problem?"

The answer is not simple.

Correct element identification, mesh density, and application of boundary conditions are all part of the following principle:

Principle requirement

" Thorough understanding of how the physical domain is *likely* to behave under the actual load/boundary conditions, and how candidate elements are *able* to behave."

General Modeling guidelines:

Include as much of the structure as possible in the model.

Do not omit part of the structure on the assumption that it does not influence the rest of the system by "being lightly stressed!"

Use finer mesh (than the one used to obtain displacements) to obtain stresses.

Keep the design detail at a level which can be easily managed for future analysis and verification.

Generate the beam models with curves and edges and not the actual cross sections.

General Modeling guidelines:
(Continued)

Eliminate minor geometric details such as fillets, chamfers, etc. They complicate the FE mesh while not influencing the analysis outcome.

If some details are necessary, include them such that they can be easily identified and suppressed prior to FE modeling and analysis.

Take advantage of "partitioning" to divide complex shapes into a series of simpler geometries.

General Modeling guidelines: *(Continued)*

If the problem involves nonlinearity or anisotropy, analyze a linear isotropic version of the problem first.

When dynamic effects are present, perform a static analysis first, using loads that approximate the dynamic effect.

In steady-state heat-transfer studies, use appropriate elements. Lower quality elements can be used for heat-transfer only.

Subsequent thermal stress analysis requires same precautions as Mechanical/Structural analysis.

Ill-Conditioning

The concept of small changes in the stiffness matrix or load vector resulting in large changes in the solution vector.

Typically occurs in the model, and regions where large differences in stiffness of the members exist, with the stiffer part supported by the softer member (element).

Sources of Error and Concept of Error estimation

One must distinguish between inherent errors in the FE process and plain "mistakes."

Mistakes include such things as entering the wrong data, use of improper units, forgetting the support or loads, etc.

Modeling errors include mathematical error, geometric error, discretization error, and numerical error (due to limitation of computing accuracy).

Sources of Error and Concept of Error estimation *(Continued)*

One source of error is the boundary mismatch between curved edges and element straight sides. This phenomenon is known as *geometric error*.

It has been shown that use of straight-sided elements may sometime cause convergence to the wrong answer in the vicinity of the curved boundary.

Curved sided isoparametric elements (such as higher order quadrilaterals) do not exhibit the convergence problem.

Geometric error may also be introduced in the application of boundary conditions.

Sources of Error and Concept of Error estimation *(Continued)*

Another source of modeling error may arise when a load is distributed over a small region.

The solution at the immediate vicinity of the load requires considerable mesh refinement. However, if the desired area is a fair distance away from the load location, coarser mesh could be sufficient.

Rule of thumb based on St. Venant's principle:

- The error is generally small when the distance (at which measurement is taken) is at least five times the width of the original distributed load.

Element-by-element stress field

This type of stress distribution is discontinuous between elements, but stresses computed from nodal averaging are continuous.

One method of judging an approximate error field is to observe the difference between element-by-element and nodal averaging quantities.

Typically, the amount of discontinuity between elements is regarded as a measure of error.

For example: Consider the stress contour indicating stress variation within a single element as follows:

Contour	Stress [Psi]
1	2511
2	2970
3	3380
4	3820

The average stress within the element is 3170 with an estimated error of 21% !

Adaptive refinement

This is the concept of attempting to minimize the relative strain energy, also called strain energy norm.

Strain Energy Norm $\eta = [\, U_E / (U_E + U) \,]^{1/2}$

$$0 \leq \eta \leq 1$$

U is based on the element-by-element stress field

U_E is the difference between U and averaged stress field.

Adaptive refinement (continued)

In adaptive solution, the value of η is determined and the mesh is successively refined to reduce η below a desired value, say 0.1 (10%).

Note that η is measured globally and does not pertain to error at any particular point.

The successive refinements tend to try to produce a mesh in which the element “error” strain energies are almost the same.

Adaptive refinement (continued)

This is based on the assumption that discontinuities in element-by-element stress field are indicative of error. However, physical discontinuities may exist which produce realistic, and often desirable, stress discontinuities.

Finally, adaptive meshing may give the analyst a false sense of security that by achieving the convergence criteria, a correct solution has been obtained.

Mesh refinement

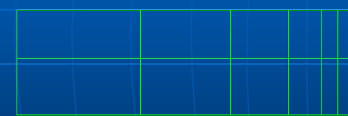
Mesh should not be refined to the extent that the size of the smallest element is considerably smaller than the largest one.

Ill-conditioning **may result depending on the ratio of element sizes.**

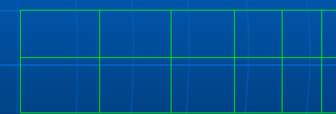
Rule of thumb: Keep the ratio to below 10:1 (linear dimensions), 100:1 (area dimensions) or 1000:1 (volume dimensions).

Elements

Meshing examples:



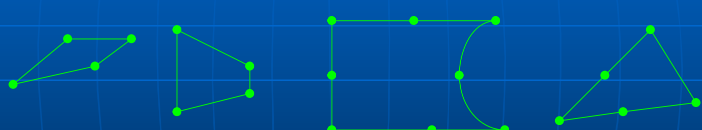
Example of poor mesh transition
(Too abrupt from coarse to fine)



Example of better mesh transition
(smoother transition from coarse to fine)

Elements *(continued)*

More examples



Examples of poorly shaped 2-D elements

Mesh refinement

h- Method:

Involves reduction in element size to achieve solution convergence.

p-method:

The order of field interpolation polynomial is successively increased until a satisfactory solution is accomplished.

r- refinement:

Involves refining the mesh by “rearranging” the existing nodes without increasing the total DOF of the problem.

Can produce limited improvement, thus not mentioned very often.

h-p refinement:

A combination of the h- and p- methods which attempts to combine best attributes of the two technique.

What is F.E. Analysis all about?

Fundamental design question:

What type of analysis is needed, and how will the outcome be related to the design performance requirements?

Element Selection

Forms the foundation of FE modeling and analysis.

Example: 4-node QUADS are preferred to CST triangular elements.

Linear formulations with successive refinements yield good results.

Avoid higher order elements unless necessary

- Example: Use of 10-node tetrahedron in ABAQUS automatic SOLID meshing is necessary for 3-D modeling involving bending and geometric irregularity.

Physical and Material properties

Make sure proper material and physical properties are used in the analysis.

A common mistake is to inadvertently use the software's defaults for the analysis.

Be Careful about units !

Boundary Conditions

A critical aspect of the modeling is the proper transformation of the actual boundary conditions to the loads/restraints set of the FE software. For example, Dynamic (Modal) analysis is very sensitive to the boundary conditions.

A model must be "fully restrained" for proper analysis. Otherwise, *rigid body* modes will cause solution failure (in Static analysis), or unexpected vibration modes (in Dynamic analysis).

Boundary Conditions (continued)

For models which are not fully restrained:

Static analysis: completely restrain at least one node. Pick a node which is not close to the area of interest (for deflection or stress investigation).

Dynamic analysis: set the minimum requested natural frequency to a small number greater than zero to avoid reporting of rigid body modes.

Solution strategy

Decide on a solution strategy which best meets the design requirements.

- Static analysis vs. Dynamic analysis
- Modal analysis vs. Shock (impact) loading

First few modes represent displacements quite well, but impact loading cannot be modeled correctly with Modal analysis solution!

Post processing

View the results with critical eyes.

Don't be impressed by color contours, and what might "seem" to be a convincing result.

Distinguish between "stepped" vs. "smooth" plots.

Check for the convergence at region(s) of interest, not just "maximum" quantity.

Common sense and good engineering judgment are far more important than the computer output.

Final Thought

" Finite Element Analysis makes a *good* engineer *great,*
and a *bad* engineer *dangerous* !"

Robert D. Cook, Professor of Mechanical Engineering
University of Wisconsin, Madison