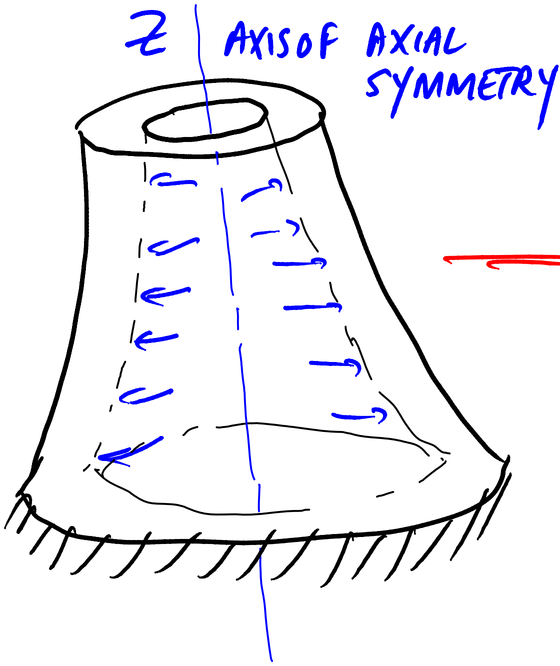
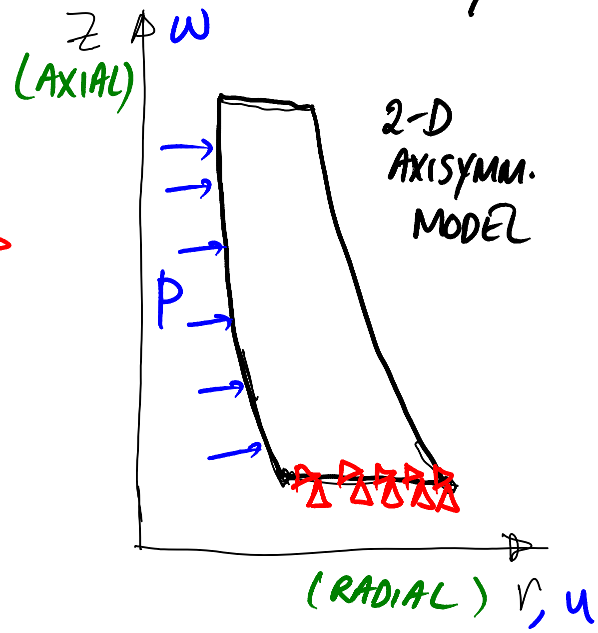
# CHAP. 9 AXISYMMETRIC ELEMENTS

SPECIAL CASE OF 3-D PROBLEM WHERE GEOMETRY, LOAD, B.C. ALL REVOLVE ABOUT AN AXIS OF SYMMETRY.




STATE OF STRAIN IN AXISYMM. PROBLEM

$$\epsilon_r = \frac{\partial u}{\partial r} \quad \text{RADIAL STRAIN} \qquad \epsilon_\theta = \frac{u}{r} \quad \text{HOOP STRAIN (TANGENTIAL)}$$

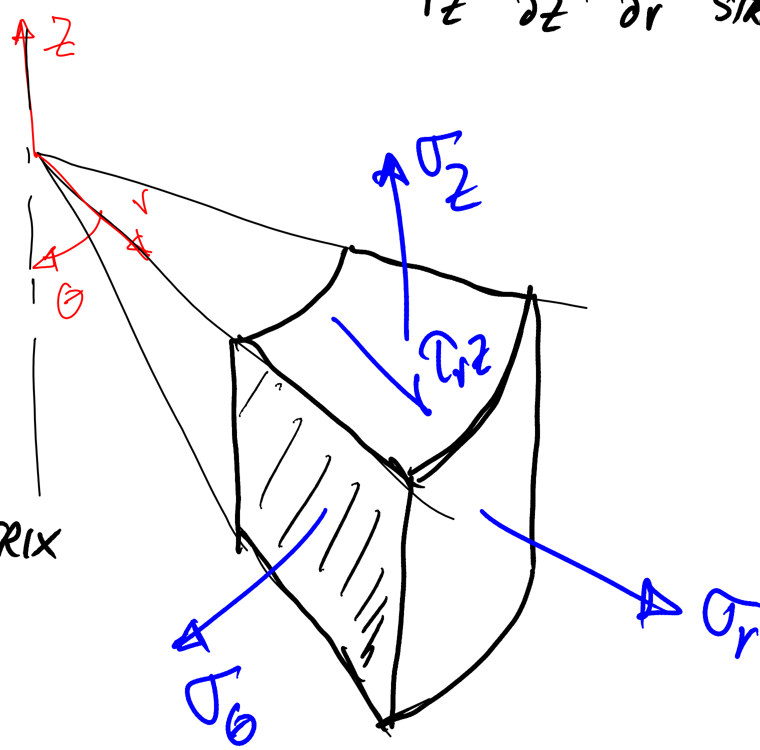
$$\epsilon_z = \frac{\partial w}{\partial z} \quad \text{AXIAL STRAIN} \qquad \gamma_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \quad \text{SHEAR STRAIN}$$

$$\{\sigma\} = [E]\{\epsilon\}$$

4X1 4X1

$[E] = [4 \times 4]$

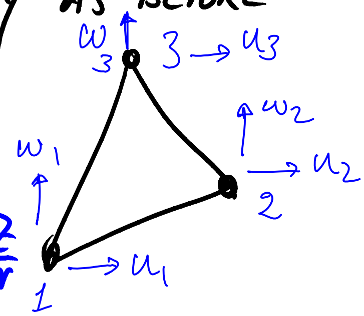
AXISYMM. CONSTITUTIVE MATRIX



## ELEMENT FORMULATION:

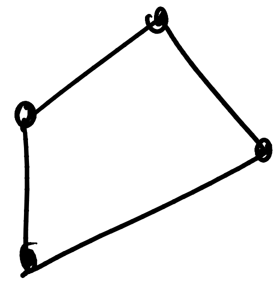
WE USE SAME 2-D ELEM. GEOMETRY AS BEFORE.

$$u(r,z) = a_1 + a_2 r + a_3 z$$
$$w(r,z) = a_4 + a_5 r + a_6 z$$




$\epsilon_r = a_2 \quad \epsilon_z = \frac{\partial w}{\partial z} = a_6 \quad \epsilon_\theta = \frac{a_1}{r} + a_2 + a_3 \frac{z}{r}$

$$u(r,z) = a_1 + a_2 r + a_3 z + a_4 rz$$
$$w(r,z) = a_5 + a_6 r + a_7 z + a_8 rz$$



$\epsilon_r = a_2 + a_4 z \quad \epsilon_z = a_7 + a_8 r$

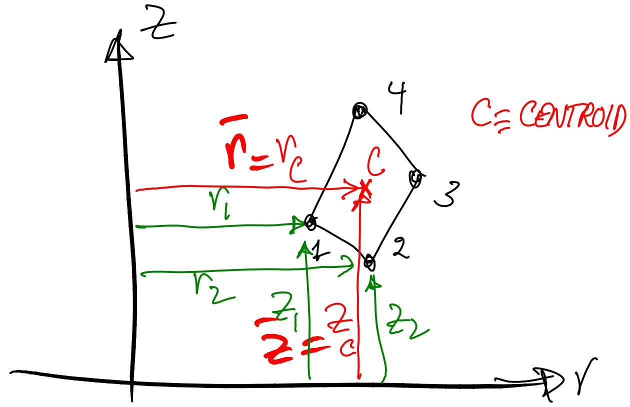
$$\epsilon_\theta = \frac{a_1}{r} + a_2 + a_3 \frac{z}{r} + a_4 z$$

$$[K] = 2\pi \int_{AREA} [B]^T [E] [B] r \, dA$$

$[B]$ AND $[B]^T$ INVOLVE MULTIPLE ORDERS (OF POWER) OF $\frac{1}{r}$ ! LEADS TO DIFFICULT NUMERICAL INTEGRATION

## APPROXIMATION METHOD:

USE CENTROIDAL COORD'S AS APPROXIMATION FOR NODAL COORD'S




$$\bar{r} = \frac{r_1 + r_2 + r_3 + r_4}{4}$$

$$\bar{z} = \frac{z_1 + z_2 + z_3 + z_4}{4}$$

CONVERTS $[B]$ INTO AN EQUIV. AVG VALUE: $[\bar{B}]$

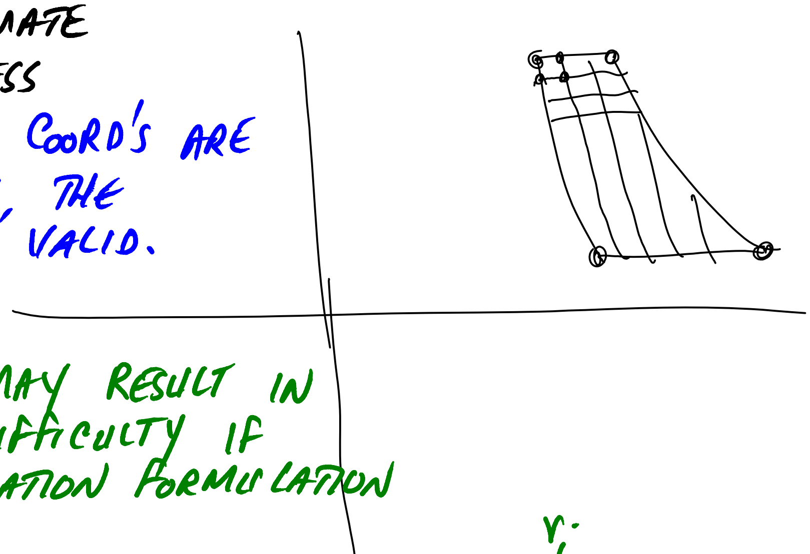
$[\bar{B}]$ IS NOW A FUNCTION OF $\bar{r}, \bar{z}$ (CONST. FOR EACH ELEM)

$$[\bar{K}] = 2\pi \bar{r} \, [\bar{B}]^T [E] [\bar{B}] \, A_e$$

$A_e$ — AREA OF ELEMENT

$[\bar{K}]$ — APPROXIMATE STIFFNESS

IF NODAL r, z COORD'S ARE CLOSE ENOUGH, THE TECHNIQUE IS VALID.



FINE MESH MAY RESULT IN NUMERICAL DIFFICULTY IF EXACT INTEGRATION FORMULATION IS USED.

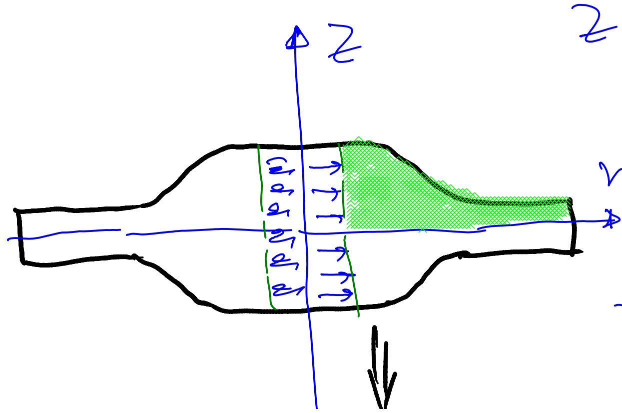
RATIO OF $\frac{r_i}{r_j}$ APPROACHES "1" AND $\int_{r_j}^{r_i} \frac{1}{r} = \ln(r)$

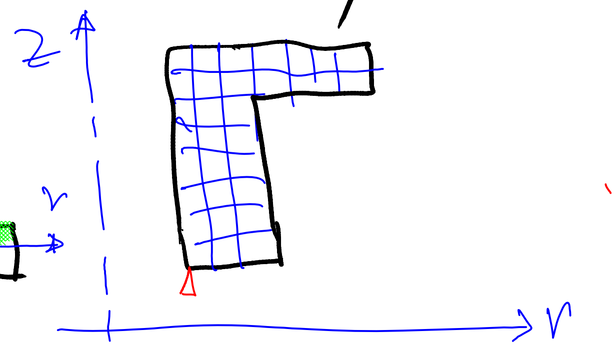
$$= \ln\left(\frac{r_i}{r_j}\right) \cong \ln(1) \cong \boxed{0}$$

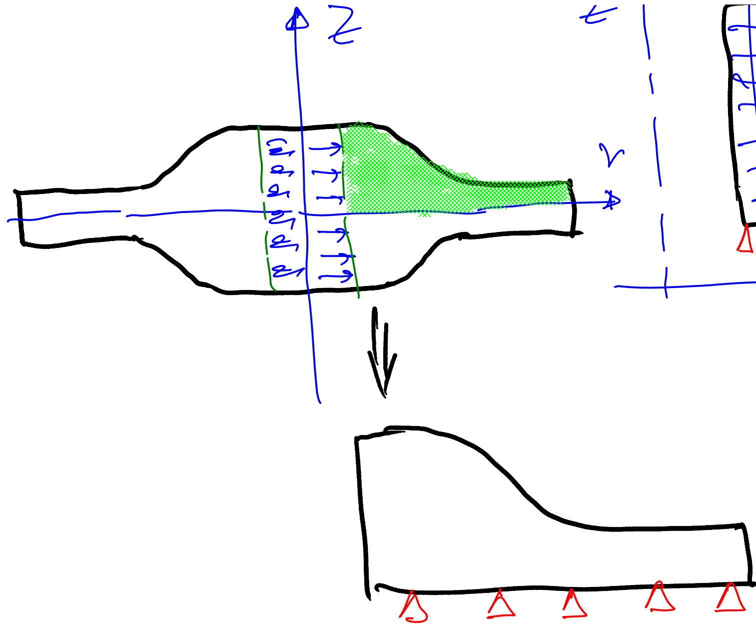
## MODELING ISSUES:

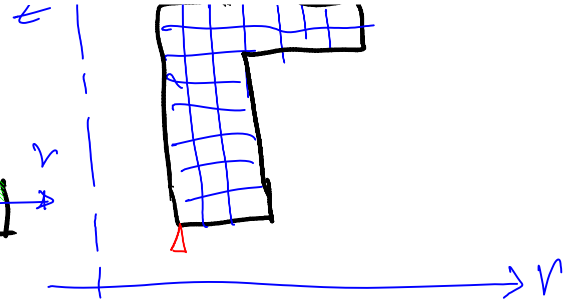
1) RADIAL D.O.F. NEED NOT BE SPECIFIED, UNLESS IT IS AN ACTUAL (PHYSICAL) RESTRAINT.

2) Z-AXIS MUST BE RESTRAINED, ATLEAST AT ONE NODE TO PREVENT RIGID BODY MOTION.



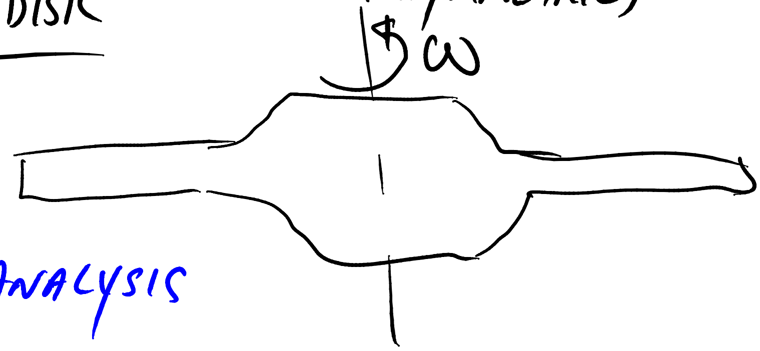






CENTRIFUGAL FORCE ACTION (SPECIAL CASE OF AXISYMMETRIC)

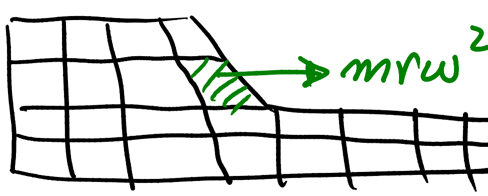
ROTATING DISK




NOT A DYNAMIC ANALYSIS

$r\omega^2$ = RADIAL ACCEL.

$mr\omega^2$ = FORCE (RADIAL)




MUST SPECIFY MATERIAL DENSITY !

$\rho r \omega^2$

IF GRAVITY IS TO BE INCLUDED:

SPECIFY VECTOR OF ACCELERATION (GRAVITY)

MAGNITUDE DIRECTION