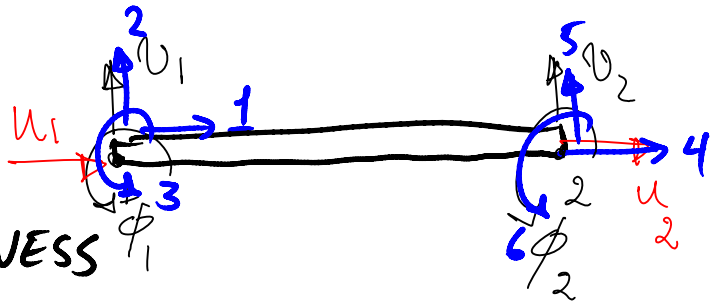


FRAME ELEMENT



INCLUDES AXIAL STIFFNESS

2-D FRAME ELEMENT (6 D.O.F.'S)

$$\hat{k} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix} & \begin{bmatrix} C_1 & 0 & 0 & -C_1 & 0 & 0 \\ 0 & 12C_2 & 6C_2L & 0 & -12C_2L & 6C_2L \\ 0 & 6C_2L & 4C_2L^2 & 0 & -6C_2L & 2C_2L^2 \\ 0 & 0 & 0 & C_1 & 0 & 0 \\ 0 & -12C_2L & -6C_2L & 0 & 12C_2L & -6C_2L \\ 0 & 6C_2L & 2C_2L^2 & 0 & -6C_2L & 4C_2L^2 \end{bmatrix} \end{matrix}$$

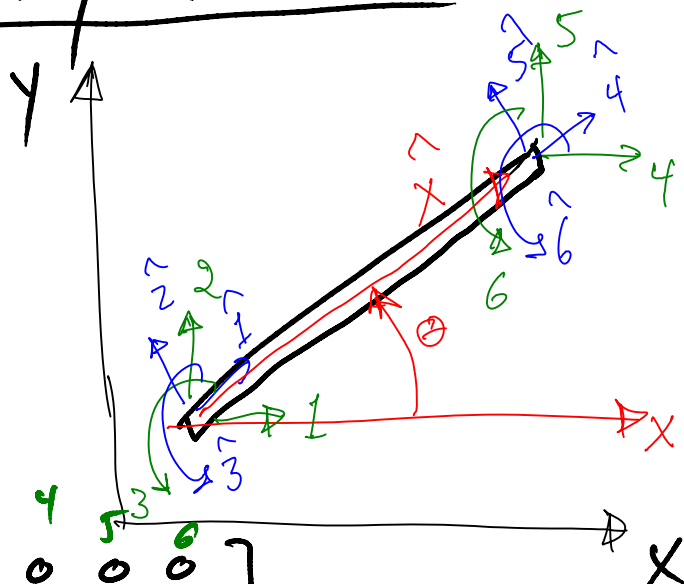
6x6

$$C_1 = \frac{AE}{L} \quad C_2 = \frac{EI}{L^3}$$

BEAMS IN ARBITRARY ORIENTATION:

$$C = \cos \theta$$

$$S = \sin \theta$$



$$[T] = \begin{bmatrix} C & S & 0 & 0 & 0 & 0 \\ -S & C & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

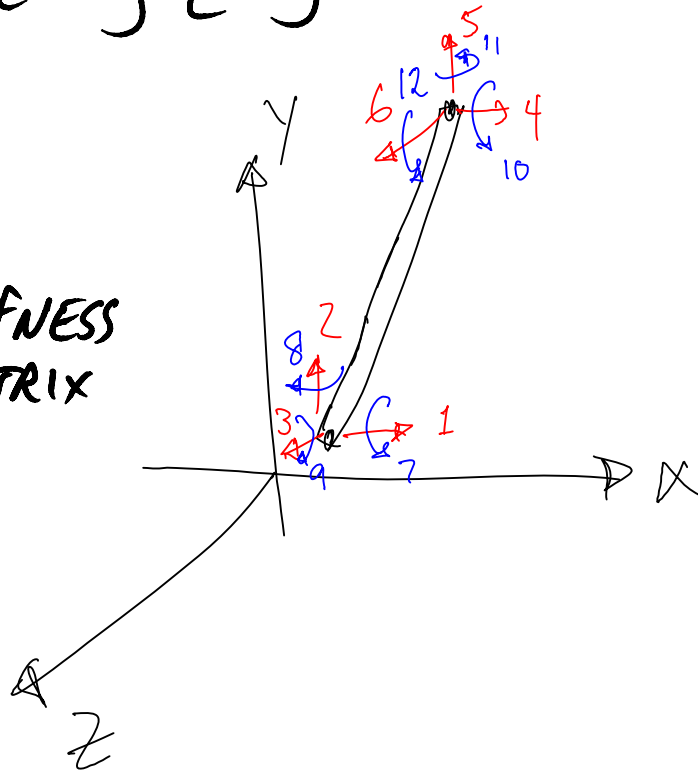
$$[U]^{-1} = \begin{bmatrix} 3 & 0 & 0 & 1 & 0 & 0 \\ 4 & 0 & 0 & 0 & c & s \\ 5 & 0 & 0 & 0 & -s & c \\ 6 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \underline{6 \times 6}$$

$$[R] = [T]^T [\hat{R}] [T]$$

6×6

3-D BEAM:

$[12 \times 12]$ STIFFNESS MATRIX

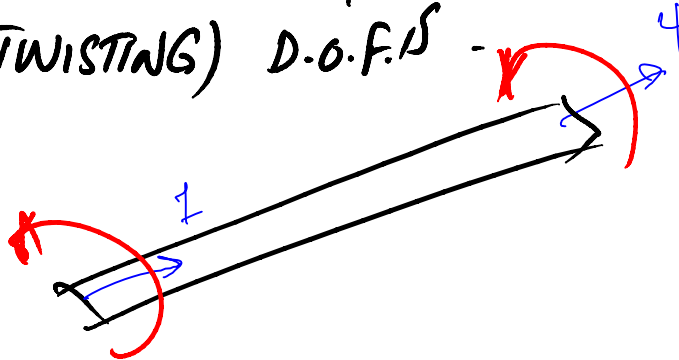


ROTATION ABOUT LOCAL 1 & 4 D.O.F'S ARE TORSIONAL (TWISTING) D.O.F'S

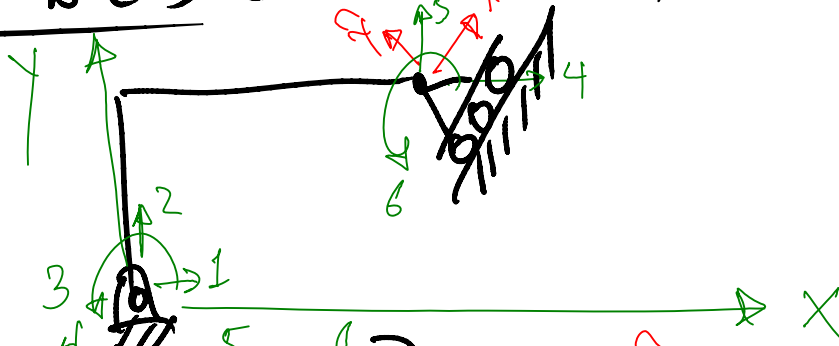
$$\theta = \frac{TL}{GJ}$$

$$T = \frac{GJ}{L} \cdot \theta$$

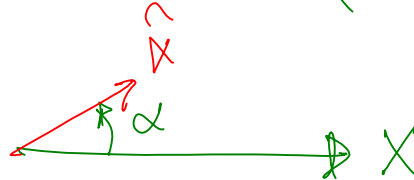
↳ TORSIONAL STIFFNESS



SKewed B.C.'s (INCLINED SUPPORT)



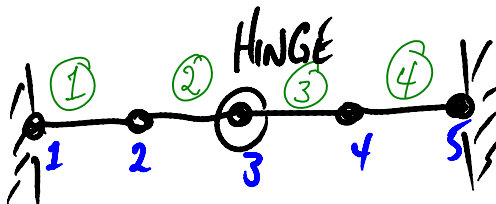
$$[t] = \begin{matrix} 4 \\ 5 \\ 6 \end{matrix} \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$\{\hat{d}\} = [T] \{d\}$$

$$[T] = \begin{bmatrix} [I] & [0] & [0] \\ [0] & [I] & [0] \\ [0] & [0] & [t] \end{bmatrix}$$

BEAM WITH HINGES:



✓ |
ZERO MOMENT
BUT NOT ZERO SLOPE!

APPLY END RELEASE TO THE NODE OF
A CERTAIN ELEMENT

ELEMENT 2 → RIGHT HAND NODE,
SET RELEASE FOR D.O.F. 3!