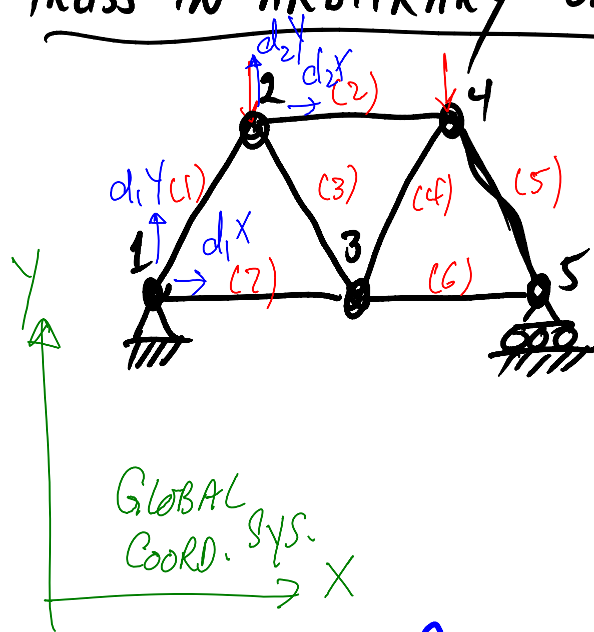
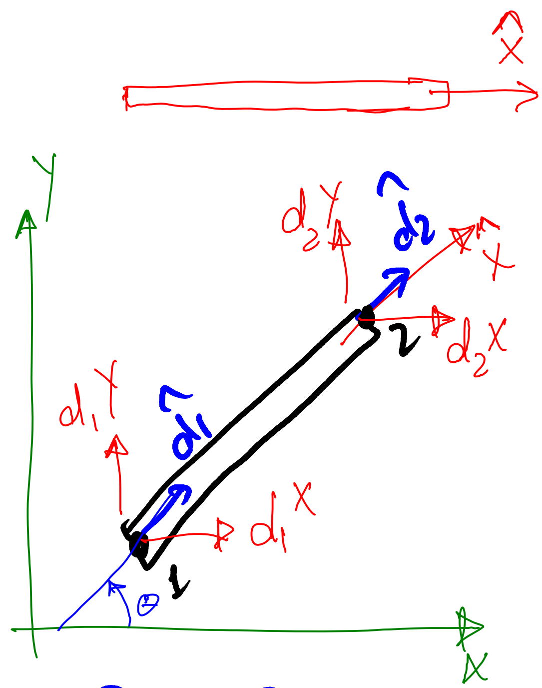
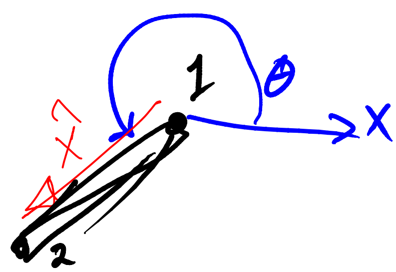
TRUSS IN ARBITRARY ORIENTATION





1) DEFINE POSITIVE $\hat{X}$ FROM NODE 1 TO 2

2) ANGLE θ IS MEASURED FROM POSITIVE X TOWARD $\hat{X}$ (CCW).



$$\hat{d}_1 = d_1{}^X \cos\theta + d_1{}^Y \sin\theta$$

$$\hat{d}_2 = d_2{}^X \cos\theta + d_2{}^Y \sin\theta$$

$$\begin{Bmatrix} \hat{d}_1 \\ \hat{d}_2 \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix} \begin{Bmatrix} d_1{}^X \\ d_1{}^Y \\ d_2{}^X \\ d_2{}^Y \end{Bmatrix}$$

$$[T] = \begin{bmatrix} C & S & 0 & 0 \\ 0 & 0 & C & S \end{bmatrix} \rightarrow \text{TRANSFORMATION MATRIX}$$

FORCE - DISP. RELATIONS:

$$(2)\quad \{f\} = [K]\{d\} \rightarrow \text{GLOBAL}$$

(2) $\{f\} = [k]\{d\} \rightarrow$ GLOBAL

(3) $\{\hat{f}\} = [\hat{k}]\{\hat{d}\} \rightarrow$ LOCAL

OBJECTIVE: RELATE $[k]$ TO $[\hat{k}]$

WE KNOW: $\{\hat{d}\} = [T]\{d\}$ (1)

SUBST. FROM (1) INTO (3) $\rightarrow$ (a) $\{\hat{f}\} = [\hat{k}][T]\{d\}$

POT. ENERGY IS INDEP. OF PATH (OR COORD. SYS).

THUS: $\frac{1}{2}\{\hat{f}\}^T\{\hat{d}\} = \frac{1}{2}\{f\}^T\{d\}$

AGAIN SUBST. FOR $\{\hat{d}\}$ FROM (1):

$$\frac{1}{2}\{\hat{f}\}^T[T]\{\cancel{d}\} = \frac{1}{2}\{f\}^T\{\cancel{d}\}$$

$$\{\hat{f}\}^T[T] = \{f\}^T \quad (4)$$

TAKE TRANSPOSE OF (4) $\rightarrow [T]^T\{\hat{f}\} = \{f\}$

PREMULTIPLY (a) BY $[T]^T$: COMPARE

$$[T]^T\{\hat{f}\} = [T]^T[\hat{k}][T]\{d\}$$

RIGHT-HANDSIDES ~~ARE EQUAL~~

$$\{f\} = [T]^T[\hat{k}][T]\{d\}$$

COMPARE

$$\{f\} = [k]\{d\}$$

$$\boxed{[k] = [T]^T[\hat{k}][T]}$$

TRANSF. OF STIFFNESS MATRIX TO GLOBAL COORD. SYS

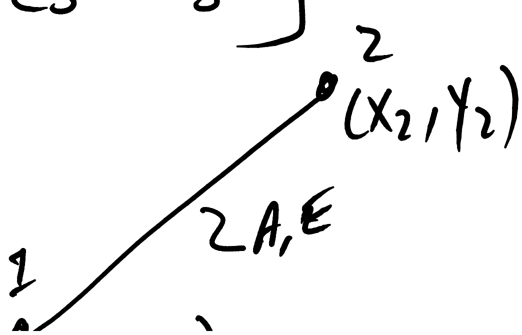
$$[\hat{k}] = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \qquad k \equiv \frac{AE}{L}$$

$$[k] = \begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix} \begin{bmatrix} k & -k \\ -k & k \end{bmatrix} \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix}$$

$$4\times 2 \qquad 2\times 2 \qquad 2\times 4$$

$$[k] = \frac{AE}{L}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

$$L = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$



$C = \frac{x_2 - x_1}{L}$ $\quad S = \frac{y_2 - y_1}{L}$ $\quad (x_1, y_1)$

$k = \frac{AE}{L}$

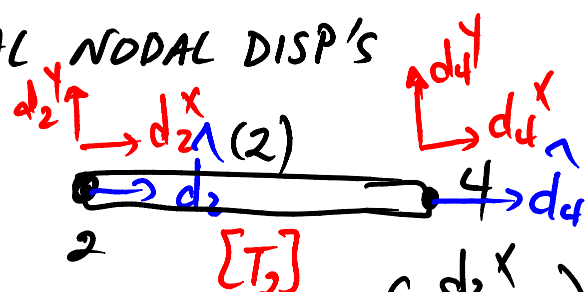
- FOR OUR ORIGINAL EXAMPLE, WE FORM $[k]$ FOR ALL ELEMENTS, COMBINE THEM INTO A GLOBAL $[10 \times 10]$ STIFFNESS MATRIX.

- APPLY B.C. (RESTRAINTS) TO END UP WITH A SET OF 7 SIMULT. EQ'NS.

- SOLVE TO OBTAIN GLOBAL DISP. COMPONENTS OF NODES.

- USE TRANSFORMATION MATRIX, IN CONJUNCTION WITH ELEM. NODAL DISP. VECTORS TO OBTAIN LOCAL NODAL DISP'S

e.g.



$$\begin{Bmatrix} \hat{d}_2 \\ \hat{d}_4 \end{Bmatrix} = \begin{bmatrix} T_2 \end{bmatrix} \begin{Bmatrix} d_2^x \\ d_2^y \\ d_4^x \\ d_4^y \end{Bmatrix}$$

$\hat{d}_4 - \hat{d}_2$

$$\epsilon_2 = \frac{\quad}{L_2} \qquad \sigma_2 = E_2 \epsilon_2$$