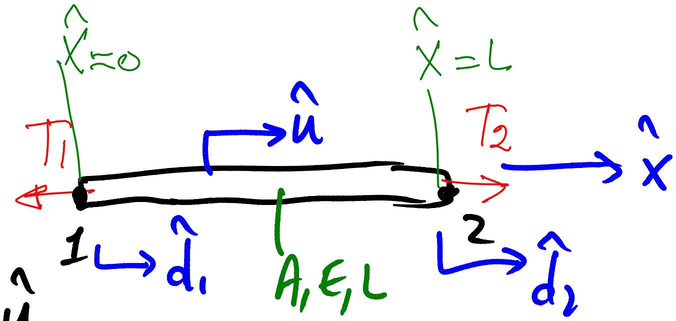
TRUSS ELEMENT:

(IN EQUILIBRIUM)

T_1, T_2 = EQUIL. FORCES



$\sigma_x = E\epsilon_x$ (a) $\quad \epsilon_x = \dfrac{d\hat{u}}{dx}$ (b)

FROM EQUIL. $\Rightarrow$ $T = A\sigma_x$ (c)

SUBST. (b) IN (a) $\rightarrow$ $\sigma_x = E\dfrac{d\hat{u}}{dx}$ (d)

SUBST. (d) IN (c) $\rightarrow$ $T = AE\dfrac{d\hat{u}}{dx}$

$T \downarrow$ CONST.

TAKE DERIVATIVE OF T $\Rightarrow$ $\boxed{\dfrac{d}{dx}\left(AE\dfrac{d\hat{u}}{dx}\right) = 0}$

GOVERNING DIFF. EQ'N
OF EQUIL. FOR TRUSS (ROD).

ASSUMED DISPLACEMENT FIELD:

e.g., $\hat{u} = a_1 + a_2\hat{x}$

ASSUME LINEAR APPROX. OF DISP. FIELD ($\hat{u}(x)$)

APPLY BOUND. CONDITIONS:

@ $\hat{x} = 0 \Rightarrow \hat{u} = \hat{d}_1 \Rightarrow \boxed{a_1 = \hat{d}_1}$

@ $\hat{x} = L \Rightarrow \hat{u} = \hat{d}_2 \Rightarrow \hat{d}_2 = \hat{d}_1 + a_2 L$

$\boxed{a_2 = \dfrac{\hat{d}_2 - \hat{d}_1}{L}}$

$\boxed{\hat{u} = \hat{d}_1 + \dfrac{\hat{d}_2 - \hat{d}_1}{L}\cdot\hat{x}}$ OR $\hat{u} = \left[1 - \dfrac{\hat{x}}{L} \quad \dfrac{1}{L}\right]\begin{Bmatrix}\hat{d}_1 \\ \hat{d}_2\end{Bmatrix}$

$$\hat{u}_2 = \frac{\quad}{L}$$

$$\boxed{\hat{u} = \hat{d}_1 + \frac{\hat{d}_2 - \hat{d}_1}{L} \cdot \hat{x}} \quad \text{OR} \quad \hat{u} = \underbrace{\begin{bmatrix} 1-\frac{\hat{x}}{L} & \frac{1}{L} \end{bmatrix}}_{\text{SHAPE FUNCTION MATRIX}} \begin{Bmatrix} \hat{d}_1 \\ \hat{d}_2 \end{Bmatrix}$$

STRAIN WITHIN ELEM. $\Rightarrow \epsilon_x = \dfrac{d\hat{u}}{d\hat{x}}$

$$\epsilon_x = \frac{\hat{d}_2 - \hat{d}_1}{L} = \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \begin{Bmatrix} \hat{d}_1 \\ \hat{d}_2 \end{Bmatrix}$$

$$\{\epsilon_x\} = \underbrace{\begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix}}_{[B] \text{ STRAIN-DISP. MATRIX}} \begin{Bmatrix} \hat{d}_1 \\ \hat{d}_2 \end{Bmatrix}$$

$$\{\epsilon_x\} = [B]\{\hat{d}\}$$

$$\{\sigma_x\} = E\{\epsilon_x\} = E[B]\{\hat{d}\}$$

DEFINITION OF STRAIN ENERGY:

$$U = \frac{1}{2}\int_{vol.} \{\sigma\}^T \{\epsilon\}\, dvol. \qquad dvol = A\,dx$$

$$\{\sigma\}^T = E\{\hat{d}\}^T [B]^T$$

$$U = \frac{1}{2}\int_0^L \underbrace{E\{\hat{d}\}^T [B]^T}_{\{\sigma\}^T} \underbrace{[B]\{\hat{d}\}}_{\{\epsilon\}} \underbrace{A\,dx}_{dvol.} \qquad ([A][B])^T = [B]^T[A]^T$$

$$U = \frac{1}{2}\{\hat{d}\}^T \left(EA\int_0^L [B]^T[B]\, dx \right) \{\hat{d}\}$$

REMEMBER

$$U = \frac{1}{2}\{x\}^T [k] \{x\}$$

FROM ANALOGY: $[k] = EA\int_0^L [B]^T[B]\,dx$

$$[k] = EA\int_0^L \underbrace{\begin{bmatrix}-\frac{1}{L}\\ \frac{1}{L}\end{bmatrix}}_{2\times 1}\underbrace{\begin{bmatrix}-\frac{1}{L} & \frac{1}{L}\end{bmatrix}}_{1\times 2}dx$$

$$[k] = EA\underbrace{\begin{bmatrix}\frac{1}{L^2} & -\frac{1}{L^2}\\ -\frac{1}{L^2} & \frac{1}{L^2}\end{bmatrix}}_{2\times 2}\int_0^L dx = L$$

$$\boxed{[k] = \frac{EA}{L}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}}$$

$$\hat{u} = \begin{bmatrix}\underbrace{1-\frac{\hat{x}}{L}}_{N_1} & \underbrace{\frac{\hat{x}}{L}}_{N_2}\end{bmatrix}\begin{Bmatrix}\hat{d}_1\\ \hat{d}_2\end{Bmatrix}$$

SHAPE FUNCTION MATRIX