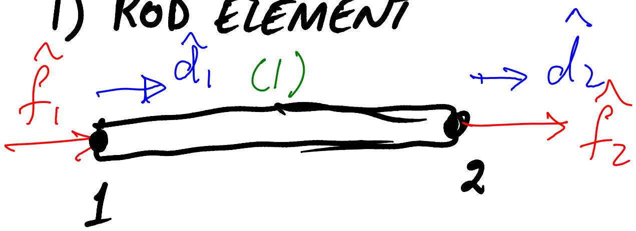
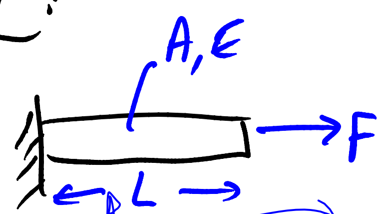
# DIRECT STIFFNESS APPROACH:

## 1) ROD ELEMENT




$k = \frac{AE}{L}$

STIFFNESS

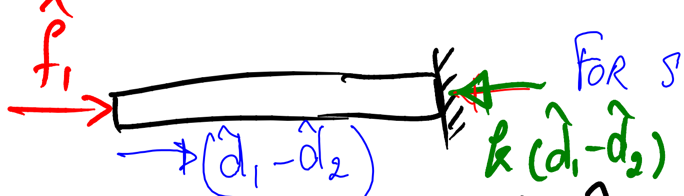
$\boxed{F = k\delta}$

$\hat{d}_1, \hat{d}_2$ = LOCAL COORD. SYS. DISP. ——→ x

$\hat{f}_1, \hat{f}_2$ = FORCES IN LOCAL COORD. SYS.

<u>QUESTION</u>: WHAT IS ELEMENT BEHAVIOR? (IN CONTEXT OF $F = k\delta$)

OBSERVER LOCATED AT NODE 2:




FOR STATIC EQUILIBRIUM

$$\Sigma F_x = 0 \Rightarrow \hat{f}_1 - k(\hat{d}_1 - \hat{d}_2) = 0$$

$$\hat{f}_1 = k\hat{d}_1 - k\hat{d}_2 \quad (1)$$

SIMILARLY IF NODE 1 IS FIXED, AND NODE 2 IS OBSERVED,


$$\Sigma F_x = 0 \Rightarrow \hat{f}_2 - k(\hat{d}_2 - \hat{d}_1) = 0$$

$$\hat{f}_2 = -k\hat{d}_1 + k\hat{d}_2 \quad (2)$$

REARRANGE (1) AND (2) IN MATRIX FORM:

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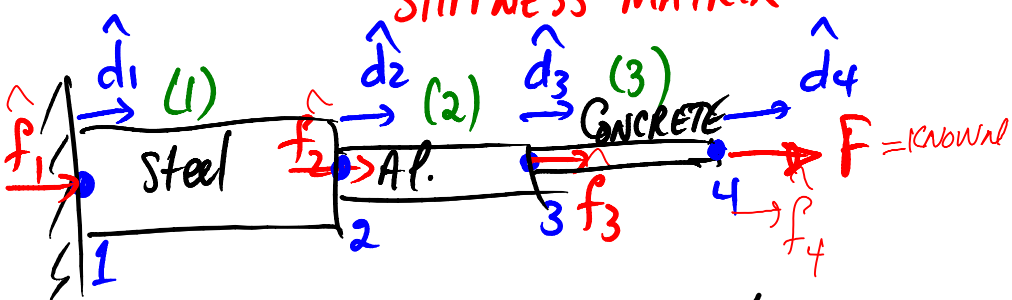
$$\begin{Bmatrix}\hat{f}_1\\ \hat{f}_2\end{Bmatrix} = \begin{bmatrix} k & -k \\ -k & k\end{bmatrix}\begin{Bmatrix}\hat{d}_1\\ \hat{d}_2\end{Bmatrix}$$

$$\{f\} = [k]\,\{d\}$$

STIFFNESS MATRIX




DETERMINE NODAL DISP's AND ELEM. STRAINS (AND STRESSES)

a) NO. OF D.O.F.'s SHOULD BE EQUAL TO TERMS OF STIFFNESS MATRIX (ROWS OR COLUMNS)

$$\begin{Bmatrix} f_1 = ?\\ f_2 = 0\\ f_3 = 0\\ f_4 = F\end{Bmatrix} = \begin{array}{c} \\ 1\\ 2\\ 3\\ 4\end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4\end{array}\\ \begin{bmatrix} k_1 & -k_1 & 0 & 0\\ -k_1 & k_1+k_2 & -k_2 & 0\\ 0 & -k_2 & k_2+k_3 & -k_3\\ 0 & 0 & -k_3 & k_3\end{bmatrix}\end{array}\begin{Bmatrix} d_1 = 0\\ d_2\\ d_3\\ d_4\end{Bmatrix}$$

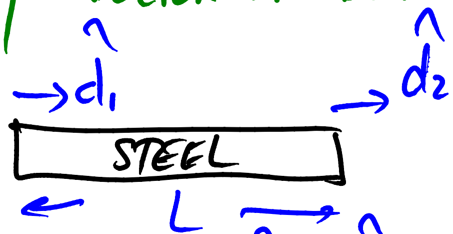
← TRI-DIAGONAL

– SYMMETRIC STIFFNESS MATRIX

SOLVE TO OBTAIN $f_1, d_2, d_3, d_4$ !

SINCE $d_1 = 0$, ELIMINATE 1ST ROW AND 1ST COLUMN.

TO FIND REACTION FORCE ($f_1$), TAKE 1ST ROW OF STIFFNESS MATRIX, MULTIPLY BY VECTOR OF DISPLACEMENT.




$$\epsilon_{STEEL} = \frac{\hat{d}_2 - \hat{d}_1}{L_{St.}} \qquad \epsilon_{Al} = \frac{\hat{d}_3 - \hat{d}_2}{L_{Al.}}$$

$$\sigma = E\epsilon$$