THERMAL (HEAT TRANSFER) ANALYSIS :

STEADY-STATE HEAT TRANSFER (TIME-INDEPENDENT)

*1) CONDUCTION → SOLID-SOLID

*2) CONVECTION → SOLID-FLUID

3) RADIATION → EMMISION OF ELECTROMAGNETIC WAVE

1) **CONDUCTION** : THERMALLY ISOTROPIC MATERIAL MAT'L

GOVERNING EQ'N : FOURIER EQ'N : $\underset{\text{HEAT FLUX}}{q_x} = -\underset{\text{THERMAL CONDUCTIVITY}}{k} \underset{\text{TEMP. GRADIENT}}{\frac{\partial T}{\partial x}}$

T ≡ TEMP. FIELD T(x)

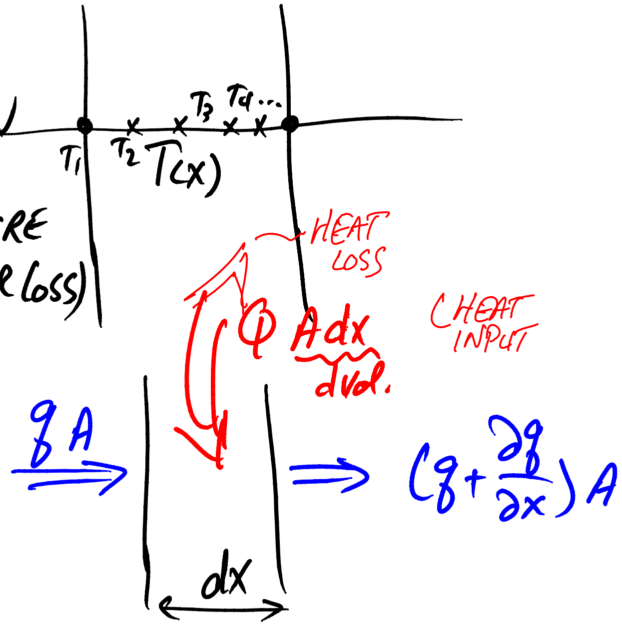
k ≡ THERMAL CONDUCTIVITY COEFF.

q ≡ HEAT FLUX (CONDUCTION)

2) **CONVECTION** : $\underset{\text{CONVECTION HEAT FLUX}}{q_h} = \underset{\text{FILM COEFF. (CONVECTIVE COEFF.)}}{h} (\overset{\text{SOLID TEMP}}{T_s} - \underset{\text{FLUID TEMP. (AMBIENT TEMP.)}}{T_\infty})$

OBJECTIVE :

DETERMINE TEMP. DISTRIBUTION GIVEN THERMAL PROPERTIES & NATURE OF HEAT INPUT (OR LOSS)



A ≡ SURFACE AREA

HEAT BALANCE CONDITION :

$qA + \Phi A dx = (q + \partial q) A$

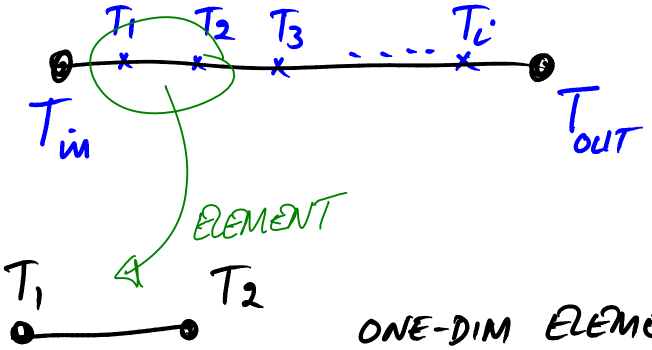
$$\cancel{q}\cancel{A} + \varphi \cancel{A}\, dx = \left(\cancel{q} + \frac{\partial q}{\partial x}\right)\cancel{A}$$

$$-\frac{\partial q}{\partial x} + \varphi\, dx = 0$$

SUBST. FOR q FROM FOURIER EQ'N, NOTING NORMAL DERIVATIVE.

$$\frac{d}{dx}\left(k\,\frac{dT}{dx}\right) + \varphi\, dx = 0$$

GOVERNING EQUIL. EQ'N TO BE SOLVED TO OBTAIN TEMP. DISTRIBUTION



ONE-DIM ELEMENT (2 D.o.F. SCALAR QUANTITY)

ASSUMED TEMPERATURE FIELD!
(SIMILAR TO ASSUMED DISP. FIELD IN STRUCTURAL ANALYSIS)

$$\{T\} = \underbrace{[N]}_{\text{SHAPE FUNCTION}}\begin{Bmatrix}T_1\\T_2\end{Bmatrix}$$

$$[N] = [N_1 \quad N_2]$$

$$N_1 = 1 - \frac{x}{L} \qquad N_2 = \frac{x}{L}$$

$$\frac{dT}{dx} = \underbrace{\frac{d[N]}{dx}}_{[B]}\begin{Bmatrix}T_1\\T_2\end{Bmatrix}$$

TEMP. DIST. VS. NODAL TEMP. MATRIX

$$[B] = \left[\frac{dN_1}{dx} \quad \frac{dN_2}{dx}\right] = \frac{1}{L}[-1 \quad 1]$$

MINIMIZATION OF POTENTIAL ENERGY:

$$\Pi = U + \Omega + \Omega + \Omega$$

h POT. ENERGY DUE TO TEMP. GRADIENT

POT. ENE. EXT. HEAT SOURCE (SINK)

POT. ENE. CONDUCT. HEAT FLUX

h POT. ENE. CONVECTIVE HEAT FLUX

CAN BE RESULT OF CONDUCTION + CONVECTION

$$\Pi = \text{COND.} + \text{CONV.}$$

$$\Pi = \underbrace{\frac{1}{2}\iiint_{Vol.}\left[k_{xx}\left(\frac{dT}{dx}\right)^2\right] dVol.} + \iint_{AREA} h[N]^T[N]\,dA$$

$$\Omega_Q = -\iiint_{Vol.} Q\cdot T\,dVol \qquad \Omega_q = \iint_{\substack{Surf.\\ \text{of } H.\ Flux}} q\,T\,ds$$

$$\Omega_h = \frac{1}{2}\iint_{\substack{Surf.\\ Convection}} h\,(T_s - T_\infty)\,dAREA$$

$$\int_0^L \underbrace{[N]^T\left(\frac{1}{dx}\left(k\frac{dT}{dx}+\varphi\right)\right)dx = 0}_{\text{GALERKIN-BUBNOV METHOD}}$$

$$\underset{\substack{\text{CONDUCTIVITY}\\ \text{MATRIX}}}{[k]} = \iiint_{V} [B]^T \underset{\substack{\downarrow\\ \text{CONDUCTIVITY}\\ \text{COEFF. MATRIX}}}{[D]} [B]\,dVol. + \iint_S h[B]^T[B]\,ds$$

$$[k] = \int_0^L \begin{bmatrix} -\frac{1}{L} \\ \frac{1}{L} \end{bmatrix} k_{xx} \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \underbrace{A\,dx}_{dVol.} + \iint_S h \begin{bmatrix} -\frac{1}{L} \\ \frac{1}{L} \end{bmatrix} \begin{bmatrix} -\frac{1}{L} & \frac{1}{L} \end{bmatrix} \underline{P}\,dx$$

$P\,dx$ = SURFACE FOR CONVECTION

$\downarrow$

PERIMETER OF ELEM.

$$[k] = \frac{\underline{A}\ \underline{k_{xx}}}{} \begin{bmatrix} 1 & -1 \end{bmatrix} +$$

$$[k] = \underbrace{\frac{\quad}{L}\begin{bmatrix} & \\ -1 & 1 \end{bmatrix}}_{[k_c]} + \underbrace{\frac{hPL}{6}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}_{[k_h]}$$

CONDUCTION — CONVECTION

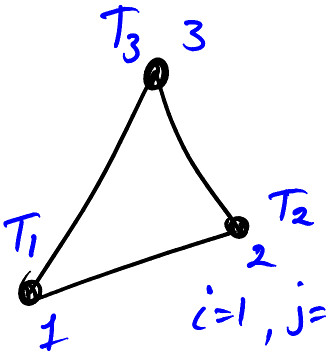
$$\overset{\text{FORE}}{\{F\}} = \overset{\text{STIFFNESS}}{[k]}\ \overset{\text{DISP.}}{\{u\}}$$

STRUCTURAL ANALYSIS

$$\underset{\text{HEAT FLUX}}{\{q\}} = \underset{\text{CONDUCTIVITY}}{[k]}\ \underset{\text{TEMP. FIELD}}{\{T\}}$$

TWO-DIM. HEAT ELEMENT

3-NODE TRIANGLE



$i=1,\ j=2,\ k=3$

$$\{T\} = [N_1 \quad N_2 \quad N_3]\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix}$$

$$N_i = \frac{1}{2A}(\underbrace{\alpha_i + \beta_i x + \gamma_i y}_{\text{LINEAR APPROX.}})$$

$$\begin{cases} \alpha_i = x_i y_k - y_i x_k \\ \beta_i = y_i - y_k \\ \gamma_i = x_k - x_j \end{cases}$$

$$[k] = [k_c] + [k_h]$$

$$= \iiint [B]^T [D] [B]\, dVol + \iint h\, [B]^T [B]\, ds$$

$$[D] \rightarrow \begin{bmatrix} k_{xx} & 0 \\ 0 & k_{yy} \end{bmatrix}$$

→ ELEMENT THICKNESS WILL BE INCORPORATED

UNITS:

$$k = \frac{W}{m \cdot {}^\circ C} \quad \text{OR} \quad \frac{BTU}{hr\text{-}{}^\circ F\text{-}ft}$$

$$Q = \frac{W}{m^3} \quad \text{OR} \quad \frac{BTU}{hr\text{-}ft^3}$$

$$q = \frac{W}{m^2} \quad \text{OR} \quad \frac{BTU}{hr\text{-}ft^2}$$

$$h = \frac{W}{m^2 \cdot {}^\circ C} \quad \text{OR} \quad \frac{BTU}{hr\text{-}ft^2 \cdot {}^\circ F}$$

HEAT TRANSFER MODELING :

1) SAME ELEMENT GEOM. AS STRUCTURAL ANALYSIS.

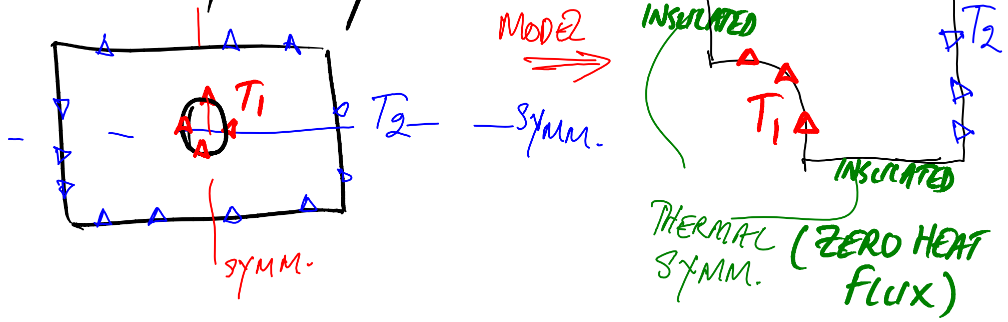
IF A SUBSEQUENT THERMAL STRESS ANALYSIS IS TO BE PERFORMED, WE WANT TO FOLLOW THE ELEMENT RULES FOR STRUCTURAL ANALYSIS.

3-NODE CST, 4-NODE TET ARE PERFECTLY FINE FOR PURE H.T. ANALYSIS BUT NOT VALID FOR THERMO/STRUCTURAL ANALYSIS.

FOR ONE-DIM. MODEL, SIMPLE TRUSS/ROD ELEM. IS FINE. IF HEAT OR TEMP. DIST. ACROSS THICKNESS IS REQUIRED, WE SHOULD USE AN EQUIV. "BEAM" FORMULATION.

"SHELL" ELEMENT FORMULATION IS USUALLY EMPLOYED FOR 2-D MODELING WHERE SURFACE HEAT FLUX (NORMAL TO SURFACE) IS PRESENT.

2) USE OF SYMMETRY :



THERMAL STRESS

THERMAL STRESS:

2 CONCEPTS LEADING TO PRESENCE OF THERMAL STRESS.

1) UNIFORM MATERIAL PROPERTY (ISOTROPIC), COUPLED WITH A TEMPERATURE GRADIENT IN THE MATERIAL

✓ 2) UNIFORM TEMP. COUPLED WITH INTERACTION OF DIFFERENT MATERIAL COEFF. OF THERMAL EXPANSION PROPERTIES.

$$\underbrace{\epsilon_0}_{\text{THERMAL STRAIN}} = \underbrace{\alpha}_{\text{COEFF. OF THERMAL EXPANSION}} \underbrace{\Delta T}_{\text{CHANGE IN TEMP.}}$$

$$\left[\frac{1}{{}^\circ F}\right] \text{ OR } \left[\frac{1}{{}^\circ C}\right]$$

$$\left[\frac{1}{{}^\circ R}\right] \text{ OR } \left[\frac{1}{{}^\circ K}\right]$$

$$\sigma_0 = E\epsilon_0$$

OVERALL STRAIN (DUE TO THERMAL & STRUCTURAL LOAD)

$$\{\sigma\} = [E]\left(\underbrace{\{\epsilon\}}_{\text{MECH. COMP.}} \mp \underbrace{\{\epsilon_0\}}_{\text{THERMAL COMP.}}\right)$$