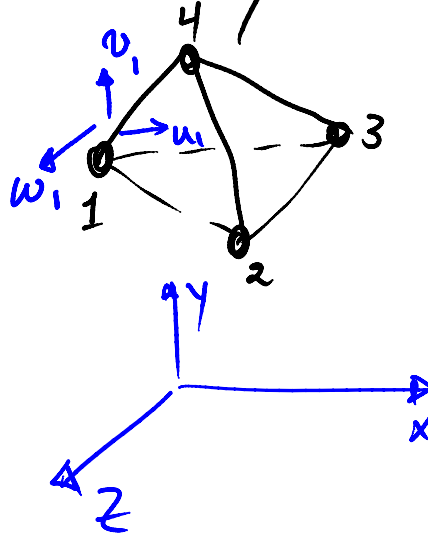


# 3-D SOLID ELEMENT:

WHEN ALL OTHER ELEMENT GEOMETRIES (& LOADS & BC'S)  
FAIL TO DESCRIBE THE PROBLEM CORRECTLY.

SIMPLEST 3-D SOLID:  
4-NODE TETRAHEDRON  
(4-NODE TET ELEMENT)

12 D.O.F.'S



$$u(x,y,z) = a_1 + a_2x + a_3y + a_4z$$

$$v(x,y,z) = a_5 + a_6x + a_7y + a_8z$$

$$w(x,y,z) = a_9 + a_{10}x + a_{11}y + a_{12}z$$

$$\left\{ \begin{aligned} \epsilon_x &= \frac{\partial u}{\partial x} = a_2 & \epsilon_y &= \frac{\partial v}{\partial y} = a_7 & \epsilon_z &= \frac{\partial w}{\partial z} = a_{12} \\ \gamma_{xy} &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = a_3 + a_6 & \gamma_{yz} &= \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = \dots & \gamma_{zx} &= \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \end{aligned} \right.$$

ALL STRAINS ARE CONST.

$$\{\epsilon\} = \{6 \times 1\} \quad \{\sigma\} = \{6 \times 1\} = \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}$$

$$\{\sigma\} = [E] \{\epsilon\}$$

$$[E] = [6 \times 6] = \frac{E}{(1+\nu)(1-2\nu)}$$

$$[k] = \iiint_{Vol} [B]^T [E] [B] dVol.$$

$$\begin{bmatrix} 1-\nu & \nu & \nu & 0 & 0 & 0 \\ & 1-\nu & \nu & 0 & 0 & 0 \\ & & 1-\nu & 0 & 0 & 0 \\ & & & \frac{1-2\nu}{2} & 0 & 0 \\ & & & & \frac{1-2\nu}{2} & 0 \\ & & & & & \frac{1-2\nu}{2} \end{bmatrix}$$

SYMM.

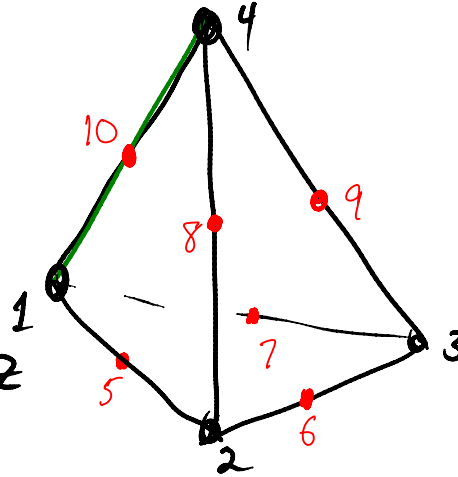
PROBLEM OF SPINNING WHEEL (UNDER LOADS) REQUIRES

PROBLEM OF SPURIOUS SHEAR (SHEAR LOCKING) BECOMES MAGNIFIED IN 4-NODE TET ELEM. THEREFORE, WE AVOID USING THIS ELEM. IN STRESS / STRAIN PROBLEMS.

UNLESS THE PORTION OF THE MODEL IS NOT SUBJECTED TO DEFORMATION.

QUADRATIC TET ELEMENT  
10-NODE TET

TOTAL OF 30 D.O.F.'S



$$u(x,y,z) = a_1 + a_2x + a_3y + a_4z + a_5x^2 + a_6y^2 + a_7z^2 + a_8xy + a_9yz + a_{10}zx$$

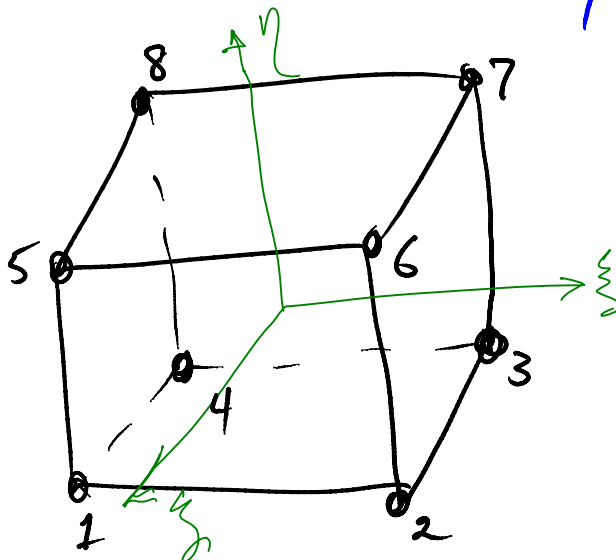
$$\epsilon_x = \frac{\partial u}{\partial x} = a_2 + 2a_5x + a_8y + a_{10}z \quad \text{— ELIMINATES SPURIOUS SHEAR}$$

— DEFAULT ELEMENT FOR FREE MESHING OF IRREGULAR GEOMETRY

SIMPLEST QUAD-EQUIV. ELEMENT IS:

8-NODE HEXAHEDRON (BRICK) ELEMENT.

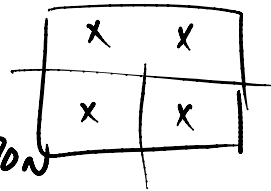
24 TOTAL D.O.F.'S



$$u(x,y,z) = a_1 + a_2x + a_3y + a_4z + a_5xy + a_6yz + a_7zx$$

$$[k] = \int_{-1}^1 \int_{-1}^1 \int_{-1}^1 [B]^T [E] [B] d\xi d\eta d\zeta + a_8 xyz$$

$\xi - \eta$  COORD. 2x2 INTEGRATION



3-D ( $\xi - \eta - \zeta$ ) COORD. 2x2x2 INTEGRATION

8- INTEGRATION POINT STRESS VALUES!

PROBLEM OF SPURIOUS SHEAR (UNDER BENDING) REMAINS.

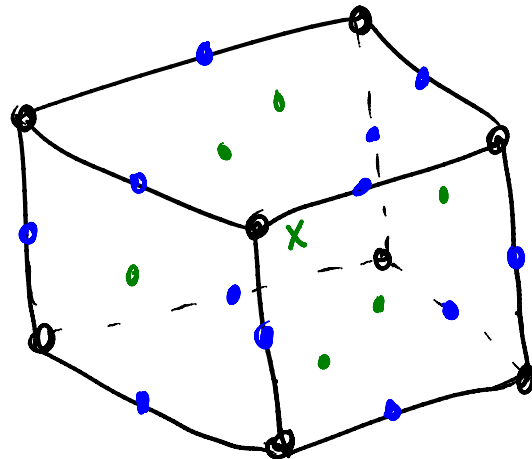
WILL USE INCOMPATIBLE FORMULATION TO OVERCOME IT.

IF NOT AVAILABLE, QUADRATIC HEXAHEDRON IS USED

20-NODE QUAD. HEX. ELEMENT

60 D.O.F.'S

SERENDIPITY ELEMENT



27-NODE QUAD. HEX. ELEMENT

(LAGRANGE ELEMENT)

81 D.O.F.'S