CS 581: Theory of Computation
Spring 2011
Mid-term exam
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This is a closed-notes, closed-book exam.

1. Context Free Grammars

   Recall that a context free grammar is a four-tuple, $G = (V, \Sigma, R, S)$.

   (a) Define the language generated by a context free grammar. Include any necessary supporting definitions.

   (b) Define the phrase "a string is generated ambiguously by grammar G". Give any additional supporting definitions required to define this concept.

   (c) Give an example of a grammar that generates a string ambiguously.

   (d) Illustrate the definition and the grammar by showing the ambiguity in your grammar.

2. Index of a language

   For each of the following languages over $\Sigma = \{a, b\}$ determine its index. For languages of finite index please give a set of pairwise distinguishable strings exactly the size of the index. For languages of infinite index give a rule that generates an infinite set of pairwise distinguishable strings (it need not be complete, but it must be infinite). Briefly justify why the strings are distinguishable.

   (a) Σ^*

   (b) a

   (c) a^*

   (d) a^*b^*

   (e) $\{a^n b^n | n \geq 0\}$

3. Myhill Nerode Theorem

   Prove that every language with finite index k is accepted by a DFA with k states. Please give a detailed argument. (This was part of a homework problem.)

4. Pumping Lemma for Context Free Languages

   Use the pumping lemma for context free languages to prove that the language of palindromes over $\{0, 1\}$ with an equal number of 0s and 1s is not context free. (Sipser 2.31)