# CS 581: Theory of Computation
## Fall 2011
## Mid-term exam
## James Hook

This is a closed-notes, closed-book exam.

1. [25 points] Regular Languages

   (a) Construct a DFA that accepts binary numbers congruent to 3 mod 4. (The string $\epsilon$ represents the number 0.) [The set of numbers congruent to 3 mod 4 are those numbers than when divided by 4 yield a remaineder of 3. They include $3, 7, 11, 15, \ldots$.]

   - [Issue] Some students gave correct rules with incorrect sketches.

   (b) Give the definition of acceptance of a string by a DFA.

   - [5 points] Correct.
   - [3–4 points] Looks like a definition
   - [1–2 points] Informal description.
   - Many students didn't seem to really understand the difference between a set of states and a sequence of states. In particular, where the definition of acceptance requires a sequence of states $r_0, r_1, \ldots, r_n$ the students used all of $Q$. Then they quantified over all elements of $Q$ rather than over all positions in the sequence $0 \leq i \leq n$.

   (c) Illustrate the definition by showing an example of a string accepted by the DFA and a string not accepted by the DFA.

   - The intended DFA was the one from part a.
   - [5 points] Illustrates definition with a positive and a negative example.
   - [3 points] Illustrates definition with one example.
   - [2 points] Two examples, but doesn't illustrate definition.
   - [1 point] One example, not illustrative.

   (d) Is your DFA minimal? Justify.

   - [5 points] Correct and justified by Myhill Nerode.
   - [3–4 points] Wrong, but justification is systematic and applying relevant general results.
   - [3–4 points] Right, but justification is ad hoc.
   - [2 points] Wrong, ad hoc.

   (e) Give a minimal DFA for the language. (If your DFA was already minimal then refer to earlier construction.)

2. [25 points] Non-regular languages.

   Let $A$ be the language over $\{a, b\}^*$ of strings containing at least as many $b$'s as $a$'s. Show that $A$ is not regular.

   - [25 points] Correct solution.
   - [20 points] Minor repair.
   - [15 points] Significant repair.
   - [5 – 10 points] Significantly flawed application of an appropriate general argument. (Pumping lemma, Myhill Nerode)
   - [5 points] Good start, but not sufficiently complete to critique.

3. [25 points] Context Free Languages

   In the homework you showed the language $A = \{x\#y | x \neq y\}$ is context free. We reviewed a solution to this problem in lecture.

   (a) [15 points] Give a grammar that generates $A$ or a PDA that recognizes $A$. Justify why the solution is correct.
   - [8 points] Construction
     - [8 points] Correct construction
     - [6 points] Incorrect construction related to problem but specific enough to critique
     - [4 points] Prose description consistent with a correct construction.
   - [7 ponits] Justification
     - [7 points] Good justification
     - [6 points] Helpful justificaiton, but bits are puzzling.
     - [3 points] Sketch of justification.

   (b) [5 points] Illustrate your solution by describing how it generates or recognizes the string 010#000.
   - [5 points] Correct; supported by calculation.
   - [3 points] A good story
   - [2 points] I say so

   (c) [5 points] Argue systematically that the string 000#000 is not accepted by your machine or generated by your grammar.
   - [5 points] Correct; explicitly considers all nondeterministic executions.
   - [3 points] A good story
   - [2 points] I say so

4. [25 points] Context Free Pumping Lemma

   Use the pumping lemma for context free languages to show that the complement of the language $A$ in the previous question is not a Context Free

Language. If you would like you may also use closure properties. In my solution I show $\bar{A} \cap ((0 \cup 1)^* \# (0 \cup 1)^*)$ is not context free, which is an equivalent problem.

- [5 points] Right quantifier structure
- [5 points] Good $s$
- [5 points] Good cases
- [10 points] Good details
  - [8 points] Minor repair
  - [5 points] Sketched with no false statements