CS 581: Theory of Computation
Fall 2011
Mid-term exam
James Hook

This is a closed-notes, closed-book exam.

1. [25 points] Regular Languages

   (a) Construct a DFA that accepts binary numbers congruent to 3 mod 4. (The string ϵ represents the number 0.) [The set of numbers congruent to 3 mod 4 are those numbers than when divided by 4 yield a remaineder of 3. They include $3, 7, 11, 15, \ldots$.]

   (b) Give the definition of acceptance of a string by a DFA.

   (c) Illustrate the definition by showing an example of a string accepted by the DFA and a string not accepted by the DFA.

   (d) Is your DFA minimal? Justify.

   (e) Give a minimal DFA for the language. (If your DFA was already minimal then refer to earlier construction.)

2. [25 points] Non-regular languages.

   Let A be the language over $\{a, b\}^*$ of strings containing at least as many b's as a's. Show that A is not regular.

3. [25 points] Context Free Languages

   In the homework you showed the language $A = \{x\#y | x \neq y\}$ is context free. We reviewed a solution to this problem in lecture.

   (a) [15 points] Give a grammar that generates A or a PDA that recognizes A. Justify why the solution is correct.

   (b) [5 points] Illustrate your solution by describing how it generates or recognizes the string 010#000.

   (c) [5 points] Argue systematically that the string 000#000 is not accepted by your machine or generated by your grammar.

4. [25 points] Context Free Pumping Lemma

   Use the pumping lemma for context free languages to show that the complement of the language A in the previous question is not a Context Free Language. If you would like you may also use closure properties. In my solution I show $\bar{A} \cap ((0 \cup 1)^*\#(0 \cup 1)^*)$ is not context free, which is an equivalent problem.