CS 581: Theory of Computation

James Hook

Final exam.

This is a closed-notes, closed-book exam.

1. Regular Languages [25 points]

   Let A be the language containing strings over the alphabet $\{a, b\}$ in which every b is followed by at least two a's.

   (a) [5 points] Give a regular expression for A.

   (b) [10 points] Determine the index of the language. Give a list of pairwise distinguishable strings representing all the equivalence classes under $\equiv_A$.

   (c) [10 points] Give a minimal DFA for A.

2. P and NP [25 points]

   (a) [5 points] Define NP.

   (b) [7 points] Illustrate the definition by showing that 3COLOR is in NP. (3COLOR was defined in your last homework, where you showed how to reduce SAT to 3COLOR.)

   (c) [5 points] Define NP Complete.

   (d) [8 points] Explain why the polynomial time reduction from 3SAT to 3COLOR shows that 3COLOR is NP Complete. (You do not need to do the reduction to solve the problem. You can assume the reduction from the last assignment.)

3. Post Correspondence Problem [25 points]

   The Post Corresondence Problem (PCP) is undecidable. The techniques used in the proof are very similar to those used in the Cook Levin theorem, showing that SAT is NP complete.

   (a) [5 points] Define the Post Correspondence Problem.

   (b) [5 points] Give an example of an instance of PCP that is solvable using no less than three "tiles".

   (c) [7 points] Sketch the reduction from A_{TM} used in the proof that PCP is undecidable, that is, show how to construct an instance of PCP that has a solution if and only if a Turing machine M accepts input w. In his proof, Sipser modifies PCP so that he can specify a leftmost tile in any solution. You may assume this same modified PCP. Please be specific about what the reduction needs to accomplish, and why it proves PCP undecidable.

   (d) [8 points] Give details of the following tiles:

      - The start tile
      - The copy tile
      - The right move tile
      - The left move tile

      For each tile describe how it is calculated from M and w, indicate how many different tiles are generated (as a function of M and w).

4. Rice's Theorem [25 points]

   (a) [5 points] State Rice's theorem. (You may use either the version I presented in email or the one in the text.)

   (b) [5 points] Give two examples of problems that you can show undecidable by applying Rice's theorem. Please explain carefully how the criteria of Rice's theorem apply to each problem.

   (c) [5 points] Give two examples of problems that cannot be shown undecidable with Rice's theorem.

   (d) [10 points] Prove Rice's theorem.