CS 581: Theory of Computation
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Final exam.

This is a closed-notes, closed-book exam.

1. (30 points)

    (a) Define a language, A, that is context free but not regular.

    (b) Show that A is context free.

    (c) Show that A is not regular.

2. (40 points)

    (a) Define a language, B, that is Turing-recognizable but not Turing-decidable.

    (b) Show that B is Turing-recognizable.

    (c) Show that B is not Turing-decidable. Do this from first principles. Do not assume the existance of another undecidable language.

    (d) Show that there is a related language that is not Turing-recognizable.

3. (30 points)

    Recall the definition of a universal programming system:

    • Lists all partial recursive functions, $\phi_0, \phi_1, \phi_2, \ldots$.

    • Includes a universal function, ϕ_{univ} with the property that:

    $$\phi_{univ}(i, x) = \phi_i(x)$$

    (a) Give an example of a universal programming system.

    (b) Sketch the definition of the universal function for your programming system. You may describe the universal function in pseudo-code for an alternative programming system if you prefer (for example, I wrote my universal function in pseudo-Haskell even though the programming system I am describing is not Haskell-like).

    In your pseudo-code you may assume helper functions that map between the numbers indexing the functions of your programming system and either concrete or abstract syntax for the programs in your system. Please state such assumptions explicitly.