CS 581: Theory of Computation
Mid-term exam
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May 3, 2005

This is a closed-book, closed-notes exam.

1. [25 points] Illustrate Language Classes

   Give examples of languages that are:

   (a) Regular,

   (b) Context Free but not Regular,

   (c) Turing-decidable but not Context Free,

   (d) Turing-recognizable but not Turing-decidable,

   (e) Not Turing-recognizable.

2. [25 points] Pumping lemma application

   Prove that the language $\{0^n 1^n | n \geq 0\}$ is not a regular language.

3. [50 points total] Non-deterministic Turing machines.

   In this problem you will sketch the proof that a non-deterministic Turing machine can be simulated by a deterministic Turing machine. Regard the machines as recognizers. Focus on the set of strings accepted (do not worry about reject states for this problem).

   Recall that both kinds of Turing machines are described as six-tuples: $M = (Q, \Gamma, \delta, q_0, q_a, q_r)$. For deterministic machines $\delta : Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R\}$. For non-deterministic machines: $\delta : Q \times \Gamma \rightarrow \mathcal{P}(Q \times \Gamma \times \{L, R\})$.

   (a) [10 points] Give the definitions of acceptance for deterministic and non-deterministic Turing machines. Define the languages recognized by the machines.

   (b) [10 points] The construction of the deterministic machine that simulates the non-deterministic machine uses a search strategy to explore the possible computations of the non-deterministic machine. Describe that strategy.

   (c) [15 points] Give the construction of the deterministic machine that simulates the non-deterministic machine. The deterministic machine can have multiple tapes (the construction in the book uses 3 tapes). Please be explicit about how this construction implements the strategy you described above.

   (d) [15 points] Argue the correctness of the construction by relating accepting computations of the non-deterministic machine to accepting computations of the simulator.