

CS 581: Theory of Computation
Spring 2011
Mid-term exam grading guide
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This is a closed-notes, closed-book exam.

1. Context Free Grammars 25 = 7 + 6 + 6 + 6

Recall that a context free grammar is a four-tuple, $G = (V, \Sigma, R, S)$.

- (a) Define the language generated by a context free grammar. Include any necessary supporting definitions.
 - Relevant discussion 2 points
 - Definition of derives 5 points
 - Derives plus set theoretic definition of $L(G)$ 7 points
- (b) Define the phrase “a string is generated ambiguously by grammar G ”. Give any additional supporting definitions required to define this concept.
 - Relevant discussion 2 points
 - Something about parse trees 4 points
 - Definition including leftmost derivation 6 points
- (c) Give an example of a grammar that generates a string ambiguously.
- (d) Illustrate the definition and the grammar by showing the ambiguity in your grammar.
 - Relevant discussion 2 points
 - Something about parse trees 4 points
 - Illustrates leftmost derivation 6 points

2. Index of a language 25 = 5 × 5

For each of the following languages over $\Sigma = \{a, b\}$ determine its index. For languages of finite index please give a set of pairwise distinguishable strings exactly the size of the index. For languages of infinite index give a rule that generates an infinite set of pairwise distinguishable strings (it need not be complete, but it must be infinite). Briefly justify why the strings are distinguishable.

- (a) Σ^*
 - Finite/Infinite 2 points
 - Set/Partition 1 or 2 additional points
 - Justification 1 additional point
- (b) a
- (c) a^*

- (d) a^*b^*
- (e) $\{a^n b^n | n \geq 0\}$

3. Myhill Nerode Theorem

Prove that every language with finite index k is accepted by a DFA with k states. Please give a detailed argument. (This was part of a homework problem.)

- Construction 10 points
- Invariant 5 points
- Proof sketch 10 points

4. Pumping Lemma for Context Free Languages

Use the pumping lemma for context free languages to prove that the language of palindromes over $\{0, 1\}$ with an equal number of 0s and 1s is not context free. (Sipser 2.31)

- Top level PL argument 10 points
- Choice of s 5 points
- Case analysis 5 points
- Details 5 points