

CS 581: Theory of Computation
James Hook
Mid-term exam

This is a closed-notes, closed-book exam.

1. [25 points] **Notation:**

In the following definitions, assume that $M = \langle Q, \Sigma, \Gamma, \delta, q_0, F \rangle$ is a PDA.

An M -*configuration* is a tripple consisting of a state, input string, and stack. That is, it is an element of $Q \times \Sigma^* \times \Gamma^*$.

The *move* relation, indicated with the symbol \vdash , is defined on configurations as follows:

$$(p, vw, ts) \vdash (q, w, rs) \quad \text{when } (q, r) \in \delta(p, v, t).$$

The relation \vdash^* is the reflexive, transitive closure of \vdash .

Using this notation, acceptance by final state and empty stack is defined:

$$\{w \mid (q_0, w, \epsilon) \vdash^* (q, \epsilon, \epsilon) \text{ and } q \in F\}$$

Problem:

- (a) [10 points] Define a PDA that recognizes palindromes (strings that read the same written forward and backward, such as “Wasilla’s all I saw” or “a man a plan a canal panama”) by final state and empty stack.
- (b) [15 points] Use the notation above to show that the machine you have constructed accepts the strings:
- i. ϵ
 - ii. 010
 - iii. 0110
2. [25 points] In lecture and in the text, a construction is given that calculates a grammar from a PDA. That construction assumes a simplified form of machine that meets three conditions: (1) it has a single accept state, (2) it accepts by empty stack, and (3) each transition either pushes a single symbol onto the stack or pops one off the stack.
- [10 points] Summarize the construction of the grammar from the machine.
 - [5 points] Modify your solution to the previous problem to meet the conditions of the construction.
 - [5 points] Apply the construction to this machine.
 - [5 points] Use the grammar calculated by the construction to generate the strings in the previous exercise.

3. [25 points] The correctness of this construction relies on the relationship that

$$A_{pq} \Rightarrow w \quad \text{iff} \quad (p, w, \epsilon) \vdash^* (q, \epsilon, \epsilon)$$

Please prove the “if” direction, that is prove that if $A_{pq} \Rightarrow w$ then $(p, w, \epsilon) \vdash^* (q, \epsilon, \epsilon)$.

4. [25 points] This problem explores adapting the construction to a less restricted form of the machine. In condition (3) above, each move is restricted to be either a push or a pop. In class we tried to apply the construction to a machine that had a move that did not change the stack. In this problem we explore extending the construction to allow transitions that do not change the stack. That is, that weaken (3) to “each transition either pushes a single symbol, pops a single symbol, or has no stack interactions.” For example, it might have $(q, \epsilon) \in \delta(p, a, \epsilon)$.
- (a) [10 points] Describe how to extend the construction to allow the weaker condition 3.
- (b) [5 points] Give an example.
- (c) [10 points] Discuss how this changes the proof in problem 3.