CS 581: Theory of Computation
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Final exam.

This is a closed-notes, closed-book exam. Each question is worth 20 points.

1. True or False.
   (a) The Regular languages are closed under intersection.
   (b) The Context free languages are closed under intersection.
   (c) The Turing decidable languages are closed under intersection.
   (d) The Turing recognizable languages are closed under intersection.
   (e) All primitive recursive functions are total computable (recursive) functions.
   (f) All total computable (recursive) functions are primitive recursive.
   (g) In a reasonable proof system all true things are provable.
   (h) In a reasonable proof system all provable things are true.
   (i) In a reasonable proof system the set of all provable statements is Turing recognizable.
   (j) In a reasonable proof system the set of all provable statements is Turing decidable.
2. Show that $\{a^n b^n | n \geq 0\}$ is of infinite index, and hence is not regular.
3. This problem focuses on the construction of a Push Down Automaton from a Context Free Grammar.
   (a) [4 points] Sketch the construction. You may use a variant of a PDA that allows for each transition to push a string of symbols.
   (b) [3 points] Apply the construction to the grammar:
   $$\begin{array}{rcl} S & \rightarrow & SS \\ & | & aSb \\ & | & \epsilon \end{array}$$
   (c) [3 points] Demonstrate the construction by showing how the PDA behaves on the string $aababb$.
   (d) [10 points] Prove that if w is generated by the grammar above then it will be accepted by the machine constructed from that grammar. Note: this is only one direction of the equivalence proof.
4. Prove that the theory $Th(\mathcal{N}, +, \times)$ is undecidable [Sipser Theorem 6.13]. You may assume that there is a primitive recursive function implementing Kleene's T-predicate. You may assume that all "min-computable" functions are representable. Please summarize the properties of these results that are critical to your argument.
5. Define $ALL_{TM} = \{\langle M\rangle | L(M) = \Sigma^*\}$. Prove that ALL_{TM} is undecidable.