CS 581: Theory of Computation
Final exam

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This is a closed-book, closed-notes, in-class exam.

Please do not consult any resources when preparing your answer. If you have questions ask the exam proctor.

1. [20 points]

   We have studied several languages and families of languages. Use the major results of class to organize the following concepts and explain their relationships. Draw a diagram showing the relationships between the language families. In an explanatory narrative briefly justify the relationships, clearly identifying when the families are known to be the same, known to be different, or suspected to be different. Place languages in the most restrictive category that you can identify.

   (a) Regular languages

   (b) Turing-recognizable languages

   (c) Context-free languages

   (d) languages recognized by a nondeterministic finite automata

   (e) languages recognized by push-down automata

   (f) Turing-decidable languages

   (g) the halting problem

   (h) The acceptance problem for Turing machines A_{TM}.

   (i) $a^n b^n c^n$

   (j) $a^n b^n$

   (k) $a^* b^*$.

   (l) The class P of polynomial time computable functions

   (m) The class NP of functions computable in nondeterministic polynomial time

   (n) The satisfiability problem for Boolean formulas

   (o) The verification problem corresponding to the satisfiability problem for Boolean formulas

   (p) The set of all sentences in the theory $\mathrm{Th}(\mathcal{N}, +, \times)$

   (q) The set of all true formulas in the theory $\mathrm{Th}(\mathcal{N}, +, \times)$

   (r) The set of all provable formulas in the theory $\mathrm{Th}(\mathcal{N}, +, \times)$ assuming a reasonable proof system in the sense of Sipser (i.e. the relation π is a proof of ϕ is decidable and the theory is sound).

2. [20 points]

   The pumping lemma for context free languages is:

   > If A is a context-free language, then there is a number p (the pumping length) where, if s is any string in A of length at least p, then s may be divided into five pieces $s = uvxyz$ satisfying the conditions:
   > (a) For each $i \geq 0$, $uv^ixy^iz \in A$,
   > (b) $|vy| > 0$, and
   > (c) $|vxy| \leq p$.

   Prove the pumping lemma for Context Free Languages.

3. [20 points]

   Without using a reduction argument or otherwise assuming the existance of another undecidable problem, show that the halting problem is undecidable.

4. [20 points]

   Sipser illustrates polynomial time mapping reducibility, or polynomial time reducibility, by showing how to reduce $3SAT$ to $CLIQUE$. The construction builds a graph from a 3-CNF formula, with a group of three nodes for every clause (called a *triple*), and a node in the group for every literal in the clause. Edges are added between all nodes except: no edges are within the same triple, and there are no edges between inconsistent values.

   Recall that a 3-CNF formula is the conjunction of a set of clauses, a clause is the disjunction of 3 literals, and a literal is a variable or its negation.

   (a) Define polynomial time reducibility.

   (b) Argue why the construction in Sipser is a polynomial time reduction.

5. [20 points]

You have been asked to evaluate a collection of advanced tools for C. Their sales claims are as follows:

Church Systems "Validation through testing." Given a program p, inputs $x_1, \ldots, x_n$ and the corresponding expected outputs $y_1, \ldots, y_n$ the CS tool reports if all tests are passed, i.e. if all inputs give the expected outputs.

Divine Systems "Valid systems halt." The DS program analysis tool does a static analysis of program p and either certifies that p halts on all inputs or gives an x such that p does not halt on input x.

Puritan Systems "The straight and narrow path leads to termination." Like the designers of the DS, the Puritans think all good programs halt. Furthermore, they fear any path that may lead to divergence. There product, the PS, checks programs for the use of control structures that might lead to non-termination. All PS certfied programs halt, but not all halting programs can be certified.

Assuming that these three vendors implement their products with computers, discuss the credibility of their respective claims. If their product cannot be implemented give an argument supporting this. If it can be implemented suggest a strategy.