Theory of Computation Homework 6

November 23, 2004

1. In Assignment 5 you experimented with two different calculi for defining functions. Although you did not show them interdefinable, you did show that all functions defined using the six schemas for partial recursive functions are represenntable in the lambda calculus.

   An important application of formal models of computational systems is Gödel's incompleteness theorem. A key step in that proof is to translate the acceptance problem for Turing machines (or some other acceptable programming system) into a sentence that is true of $\langle M \rangle$ and w if and only if M accepts w. For example, Sipser gives such a sentence in Lemma 6.12.

   For the Turing machine model we have discussed a predicate, T, that can be used to build this sentence. The definition of T is: $T(\langle M \rangle, w, x)$ is true if x is a halting computation history of machine M on input w.

   Given T, Sipser's sentence $\exists x\ \phi_{M,w}$ can be formulated as $\exists x\ T(\langle M \rangle, w, x)$.

   Argue that the T predicate is representable in $\mathrm{Th}(\mathcal{N}, +, \times)$. Do not give a formal proof, but please site specific results as appropriate. You may site results from Sipser, from Machtey and Young, or from other resources. In grading this exercise I am looking for evidence that you understand the role of representability in the incompleteness results. I am not looking for detailed mastery of the calculi.