Assignment 6

Due: November 17th

Problem 1 Give an informal description for a Turing machine that decides $\{w \mid w$ contains twice as many 0s as 1s $\}$

Problem 2 Show that the Turing-decidable languages are closed under,

a) union

b) intersection

c) complement

d) set difference

Problem 3 (Sipser 3.11) Show that "doubly infinite tape" Turing machines recognize the class of Turing-recognizable languages.

Note: A "doubly infinite tape" Turing machine is a Turing machine variant where the tape is infinite to the left as well as to the right. Computation is defined as usual except that the head will never reach the left-end of the tape. Just like ordinary Turing machines, the tape is initially filled with blanks except for the portion that contains input.

Problem 4 (Sipper 3.18) Show that a language is decidable if and only if some enumerator enumerates the language in standard-string order.

Note 1: An enumerator is a Turing machine that is attached to a printer (which can be thought of as a write-only tape). Symbols on the printer tape are from $\Sigma \cup \{\#\}$ for some special symbol $\#$, used as a delimiter. An enumerator E enumerates a string w by step s if there is a sequence of configurations of length s that leaves $\#w\#$ on the printer tape. See Theorem 3.21 in Sipser and the accompanying proof for an example of an argument using enumerators.

Note 2: "Standard-string order", also called "lexicographic order", orders strings based first on length and then *alphabetically* based on an ordering of symbols.

More formally, let $<_{lex}$ be the lexicographic ordering operator.

$$\begin{aligned} \text{Define} \quad & w <_{lex} v \\ & \text{if } |w| < |v| \\ \text{or else if } & |w| = |v| \text{ and } w_i <_{ord} v_i \text{ for the first } i \text{ at which } w_i \text{ and } v_i \text{ differ} \end{aligned}$$

where $w = w_1 w_2 ... w_k$ and $v = v_1 v_2 ... v_k$ and $<_{ord}$ is an ordering on Σ (like $a <_{ord} b <_{ord} c$ for $\Sigma = \{a, b, c\}$).