CS 311: Computational Structures
Problem Set 5

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Due: November 20, 2014

1. This problem is based on Sipser 2.22; I discussed this briefly in lecture. I changed my mind about assigning it.

   To explain the hint, I introduce a notation $x.i$ for the ith character of string x. I did not specify if this is 0 based or 1 based; you may use either convention provided you are consistent. If x is the string 0110 then the 0-based interpretation would be that $x.0 = 0$, $x.1 = 1$, $x.2 = 1$ and $x.3 = 0$.

   (a) Show that the language $A = \{x\#y \mid |x| \neq |y|\}$ is context free. (That is, strings of different lengths.)

   (b) Show that the language $B = \{x\#y \mid x.i = 0 \wedge y.i = 1 \text{ for some position } i\}$ is context free, where $x.i$ is the ith symbol of x.

   (c) Show that $\{x\#y \mid x \neq y\} = A \cup B \cup C$, where $C = \{x\#y \mid x.i = 1 \wedge y.i = 0 \text{ for some position } i\}$.

   (d) Conclude $D = \{x\#y \mid x \neq y\}$ is a context free language.

   (e) Observe that $\bar{D} \cap (0 \cup 1)^*\#(0 \cup 1)^* = \{x\#y \mid x = y\}$, a language known to not be context free. Discuss why this is evidence that the Context Free Languages are not closed under complement.

2. Sipser 2.30 (d) [Context Free language Pumping Lemma]

3. Sipser 3.3 [Modification of simulation of non-deterministic TM]

4. Sipser 3.15 (d) (e) [Closure properties of Turing-decidable languages]

5. Sipser 3.16 (b) (c) (d) [Closure properties of Turing-recognizable languages]