CS 311: Computational Structures

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5 GNFA Example in Detail

This note illustrates the application of the construction in Sipser's proof of Lemma 1.60 to the modulo 3 counter DFA.

In lecture we have shown the DFA that recognizes binary numbers modulo 3. $M = (\{0,1,2\},\{0,1\},\delta,0,\{2\})$, where δ is given by the table:

$$\begin{array}{rcl}
\delta(0,0) & = & 0 \\
\delta(0,1) & = & 1 \\
\delta(1,0) & = & 2 \\
\delta(1,1) & = & 0 \\
\delta(2,0) & = & 1 \\
\delta(2,1) & = & 2
\end{array}$$

Convert this to a GNFA by adding states s and a, and labeling all transitions with regular expressions. The GNFA can be seen by this table, where each row is the "from" state and each column is the "to" state:

δ	0	1	2	a
s	ϵ	$\emptyset$	$\emptyset$	$\emptyset$
0	0	1	$\emptyset$	$\emptyset$
1	1	$\emptyset$	0	$\emptyset$
2	$\emptyset$	0	1	ϵ

To "rip" state 0 out of this machine, we calculate the following δ':

δ'	1	2	a
s	$(\delta(s,0))(\delta(0,0))^*(\delta(0,1)) \cup (\delta(s,1))$	$(\delta(s,0))(\delta(0,0))^*(\delta(0,2)) \cup (\delta(s,2))$	$(\delta(s,0))(\delta(0,0))^*(\delta(0,a)) \cup (\delta(s,a))$
1	$(\delta(1,0))(\delta(0,0))^*(\delta(0,1)) \cup (\delta(1,1))$	$(\delta(1,0))(\delta(0,0))^*(\delta(0,2)) \cup (\delta(1,2))$	$(\delta(1,0))(\delta(0,0))^*(\delta(0,a)) \cup (\delta(1,a))$
2	$(\delta(2,0))(\delta(0,0))^*(\delta(0,1)) \cup (\delta(2,1))$	$(\delta(2,0))(\delta(0,0))^*(\delta(0,2)) \cup (\delta(2,2))$	$(\delta(2,0))(\delta(0,0))^*(\delta(0,a)) \cup (\delta(2,a))$

δ'	1	2	a
s	$\epsilon 0^* 1 \cup \emptyset$	$\epsilon 0^* \emptyset \cup \emptyset$	$\epsilon 0^* \emptyset \cup \emptyset$
1	$10^* 1 \cup \emptyset$	$10^* \emptyset \cup 0$	$10^* \emptyset \cup \emptyset$
2	$\emptyset 0^* 1 \cup 0$	$\emptyset 0^* \emptyset \cup 1$	$\emptyset 0^* \emptyset \cup \epsilon$

δ'	1	2	a
s	0^*1	$\emptyset$	$\emptyset$
1	10^*1	0	$\emptyset$
2	0	1	ϵ

Next, rip state 2 out:

δ''	1	a
s	$(\delta'(s,2))(\delta'(2,2))^*(\delta'(2,1)) \cup (\delta'(s,1))$	$(\delta'(s,2))(\delta'(2,2))^*(\delta'(2,a)) \cup (\delta'(s,a))$
1	$(\delta'(1,2))(\delta'(2,2))^*(\delta'(2,1)) \cup (\delta'(1,1))$	$(\delta'(1,2))(\delta'(2,2))^*(\delta'(2,a)) \cup (\delta'(1,a))$

δ''	1	a
s	$\emptyset 1^* 0 \cup 0^* 1$	$\emptyset 1^* \epsilon \cup \emptyset$
1	$01^* 0 \cup 10^* 1$	$01^* \epsilon \cup \emptyset$

δ''	1	a
s	$0^* 1$	$\emptyset$
1	$01^* 0 \cup 10^* 1$	01^*

Finally, we rip out state 1, leaving the single transition:

$$(\delta''(s,1))(\delta''(1,1))^*(\delta''(1,a)) \cup (\delta''(s,a))$$

Which becomes:

$$0^* 1(01^* 0 \cup 10^* 1)^* 01^* \cup \emptyset$$

Or simply:

$$0^* 1(01^* 0 \cup 10^* 1)^* 01^*$$

The result is a regular expression generating the set of binary numbers that are congruent to 2 modulo 3.

Exercise 5.1 *What if we ripped the states in a different order?*