The electric field of a uniform plane wave propagating in free space is,

$$\tilde{E} = (\hat{x} + i\hat{y})\, 30\, e^{-i\frac{\pi}{6}z} \quad (V/m)$$

Specify the modulus and direction of the electric field intensity at the $z=0$ plane at $t = 0, 5, 10$ ns.

$$\tilde{E}(z) = \hat{x}\,\tilde{E}_x(z) + \hat{y}\,\tilde{E}_y(z) = \hat{x}\, 30\, e^{-i\frac{\pi}{6}z} + i\hat{y}\, 30\, e^{-i\frac{\pi}{6}z}$$

$$= \hat{x}\, 30 e^{-i\frac{\pi}{6}z} + \hat{y}\, 30 e^{-i\frac{\pi}{6}z}\, e^{i\frac{\pi}{2}}$$

$$= \hat{x}\, 30 e^{-i\frac{\pi}{6}z} + \hat{y}\, 30\, e^{-i\frac{\pi}{6}z + i\frac{\pi}{2}}$$

$$= \hat{x}\, 30\, e^{-i\frac{\pi}{6}z} + \hat{y}\, 30\, e^{i(\frac{\pi}{2} - \frac{\pi}{6}z)} \qquad E_{x0} = a_x, \quad E_{y0} = a_y e^{i\delta}$$

$$\bar{E}(z,t) = \mathrm{Re}\{\tilde{E}(z)\, e^{i\omega t}\}$$

$$= \mathrm{Re}\{(\hat{x}\, 30 e^{-i\frac{\pi}{6}z} + \hat{y}\, 30 e^{i(\frac{\pi}{2} - \frac{\pi}{6}z)})\, e^{i\omega t}\}$$

$$= 30\cos(\omega t - \tfrac{\pi}{6}z) + 30\cos(\omega t - \tfrac{\pi}{6}z + \tfrac{\pi}{2})$$

$$= 30\left(\cos(\omega t - \tfrac{\pi}{6}z) - \sin(\omega t - \tfrac{\pi}{6}z)\right) \quad (V/m) \qquad : k = \pi/6$$

Intensity of $E(z,t)$ is given by the modulus $|E(z,t)|$.

$$|E(z,t)| = \sqrt{(E_x^2(z,t) + E_y^2(z,t))}$$

$$= \sqrt{(a_x^2 \cos^2(\omega t - kz) + a_y^2 \cos^2(\omega t - kz + \delta))}$$

$$= a = \underline{\underline{30}}$$

$$\psi(z,t) = \arctan\left(\frac{E_y(z,t)}{E_x(z,t)}\right) = \tan^{-1}\left(\frac{-30\sin(\omega t - \frac{\pi}{6}z)}{30\cos(\omega t - \frac{\pi}{6}z)}\right) = -(\omega t - \tfrac{\pi}{6}z)$$

$$K = \frac{2\pi}{\lambda} \qquad f = 2\pi\omega = \frac{c}{\lambda} = \frac{cK}{2\pi} = \frac{c\,(\pi/6)}{2\pi} = 25 \text{ MHz}$$

$$\lambda = \frac{2\pi}{K} \qquad \omega = 50\pi \times 10^6$$

@ $z=0$

$$\psi(0,t) = -\omega t = -(50\pi \times 10^6)\, t \qquad \begin{cases} t = 0, & 0 \\ t = 5\text{ns}, & -0.25\pi = -45^\circ \\ t = 10\text{ns}, & -0.5\pi = -90^\circ \end{cases}$$