Problem 7.3 Ulaby

The electric field phasor of a uniform plane wave is given by $\tilde{E} = \hat{y}\, 10 e^{j0.2z}$ (V/m). If the phase velocity of the wave is 1.5×10^8 (m/s) and $\mu_r = 2.4$, find;

a) $k = 2\pi/\lambda$

$$\lambda = 2\pi/k = 2\pi/0.2 = 10\pi \simeq 31.42 \text{ m}$$

b) $$f = v_p/\lambda = 1.5 \times 10^8 / 10\pi = \frac{0.15}{\pi} \times 10^8 = 4.77 \times 10^6 \text{ Hz}$$

c) $$v_p = 1/\sqrt{(\mu\varepsilon)} = 1/\sqrt{(\mu_0\mu_r\,\varepsilon_0\varepsilon_r)} = \frac{1}{\sqrt{\mu_0\varepsilon_0}}\,\frac{1}{\sqrt{\mu_r\varepsilon_r}} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}$$

$$\sqrt{\varepsilon_r} = \frac{c}{\sqrt{(\mu_r)}\,v_p}$$

$$\varepsilon_r = \left(\frac{c}{v_p}\right)^2 \cdot \frac{1}{\mu_r} = \left(\frac{3}{1.5}\right)^2 \frac{1}{2.4} = \frac{4}{2.4} \simeq 1.667$$

d) $$\bar{H}(z,t) = \mathrm{Re}\{\tilde{H}(z)\, e^{j\omega t}\}$$

$$\tilde{H}(z) = \frac{1}{\eta}\left(-\hat{z} \times \tilde{E}(z)\right) = \frac{1}{\eta}\left(-z \times \hat{y}\, 10 e^{j0.2z}\right) = \hat{x}\,\frac{10 e^{j0.2z}}{\eta}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0\mu_r}{\varepsilon_0\varepsilon_r}} = \sqrt{\left(\frac{4\pi\times10^{-7}\cdot 2.4}{8.85\times10^{-12}\times1.67}\right)} = 451.625\ \Omega$$

$$\tilde{H}(z) = \hat{x}\,\frac{10 e^{j0.2z}}{451.625} = \hat{x}\, 22.14\, e^{j0.2z} \text{ (mA/m)}$$

$$\bar{H}(z,t) = \mathrm{Re}\{\hat{x}\, 22.14\, e^{j0.2z}\, e^{j\omega t}\} \qquad \omega = 2\pi f = 9.54\pi\times10^6 \text{ rad/s}$$

$$= 22.14\cos(9.54\pi\times10^6 t - 0.2z) \text{ (mA/m)}$$