6.3 A coil of 100 turns of wire wrapped around a square frame of sides 0.25 m. The coil is centered at the origin with each side parallel to the x- or y-axis.

Find the induced emf across the open-circuited ends of the coil if the magnetic field is given by:

a) $\bar{B} = \hat{z}\, 20 e^{-3t}$ (T)

$V_{emf} = V^{tr}_{emf} + V^{m}_{emf}$ Note: coil is not moving therefore $V^{m}_{emf} = 0$.

$V_{emf} = V^{tr}_{emf} + 0$

$$V_{emf} = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}\int_s \bar{B}\cdot d\bar{s} = -N\frac{d}{dt}\int_{-y}^{y}\int_{-x}^{x} \bar{B}\cdot\hat{z}\, dx\, dy$$

$$= -100\frac{d}{dt}\int_{-0.125}^{0.125}\int_{-0.125}^{0.125} \hat{z}\, 20 e^{-3t}\cdot\hat{z}\, dx\, dy \qquad \text{Note: } \hat{z}\cdot\hat{z} = 1$$

$$= -100\frac{d}{dt}\int_{-0.125}^{0.125}\int_{-0.125}^{0.125} 20 e^{-3t}\, dx\, dy$$

$$= -100\frac{d}{dt}\int_{-0.125}^{0.125} 20x e^{-3t}\Big|_{-0.125}^{0.125} dy$$

$$= -100\frac{d}{dt}\int_{-0.125}^{0.125} 0.5 e^{-3t}\, dy$$

$$= -100\frac{d}{dt}\, 0.5y e^{-3t}\Big|_{-0.125}^{0.125}$$

$$= -100\frac{d}{dt}\, 0.125 e^{-3t}$$

$$= -100\cdot(-0.375 e^{-3t})$$

$$= 375 e^{-3t} \text{ (V)}$$

b) $\overline{B} = \hat{z}\, 20 \cos(x) \cos(10^3 t)$ (T)

$$V_{emf} = -100 \frac{d}{dt} 20 \cos(10^3 t) \int_{-0.125}^{0.125} \int_{-0.125}^{0.125} \cos(x)\, dx\, dy$$

$$= -100 \frac{d}{dt} 20 \cos(10^3 t) \int_{-0.125}^{0.125} \sin(x) \Big|_{-0.125}^{0.125} dy$$

$$= -100 \frac{d}{dt} 20 \cos(10^3 t) (0.2493 y) \Big|_{-0.125}^{0.125}$$

$$= -100 \frac{d}{dt} 20 \cos(10^3 t) (0.0623)$$

$$= -100 (20)(0.0623)(-10^3) \sin(10^3 t)$$

$$= 124.6 \sin(10^3 t) \quad (kV)$$

c) $\overline{B} = \hat{z}\, 20 \cos(x) \sin(2y) \cos(10^3 t)$ (T)

$$V_{emf} = -100 \frac{d}{dt} 20 \cos(10^3 t) \int_{-0.125}^{0.125} \int_{-0.125}^{0.125} \cos(x) \sin(2y)\, dx\, dy$$

$$= -100 (20)(-10^3) \sin(10^3 t) \int_{-0.125}^{0.125} \sin(x) \sin(2y) \Big|_{0.125}^{0.125} dy$$

$$= -100 (20)(-10^3) \sin(10^3 t)(0.2493) \int_{-0.125}^{0.125} \sin(2y)\, dy$$

$$= -100 (20)(-10^3) \sin(10^3 t)(0.2493)(-\tfrac{1}{2} \cos(2y)) \Big|_{-0.125}^{0.125}$$

$$= -100 (20)(-10^3) \sin(10^3 t)(0.2493)(-0.484 - (-0.484))$$

$$= -100 (20)(-10^3) \sin(10^3 t)(0.2493)(0)$$

$$= 0 \quad (V)$$