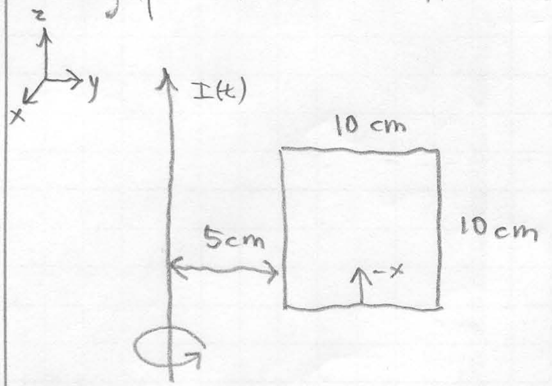
6.6 The square loop is coplanar with a long, straight wire carrying a current

$I(t) = 5\cos(2\pi \times 10^4 t)$ (A)

a) Determine the emf induced across a small gap created in the loop.



Φ in the $\hat{\phi}$ direction ACW

$$\bar{B} = \hat{\phi}\,\mu_0 \frac{I}{2\pi r} = -\hat{x}\,\mu_0 \frac{I}{2\pi y}$$

$$V_{emf} = -\frac{d\Phi}{dt} = -\frac{d}{dt}\int_s \bar{B}\cdot ds = V^{tr}_{emf} + \cancel{V^{m}_{emf}}^{\,0} \quad \text{(Faraday's law)}$$

$$= -\frac{d}{dt}\int_s \bar{B}\cdot ds$$

Note cart. $d\bar{s} = \hat{x}\,dz\,dy$

integrate wrt dy because field is homogeneous in z.

$$= -\int_{0.05}^{0.15} -\hat{x}\,\mu_0 \frac{I_0}{2\pi y}\cdot(-\hat{x}\,0.1)\,dy$$

$$= -\frac{d}{dt}\left[\mu_0 \frac{I_0\cdot 0.1}{2\pi}\int_{0.05}^{0.15} \frac{\cos wt}{y}\,dy\right]$$

$$= -\frac{d}{dt}\left[\mu_0 \frac{I_0\cdot 0.1}{2\pi}\cos wt\,\ln(0.15/0.05)\right]$$

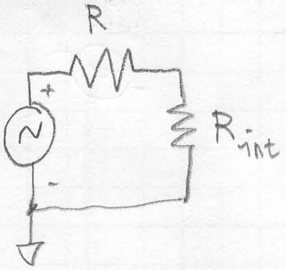
$$= \frac{\mu_0 I_0\cdot 0.1}{2\pi}\sin(wt)\ln(15/5) \quad (V)$$

$I_0 = 5$, $w = 2\pi f$, $f = 10$ KHz

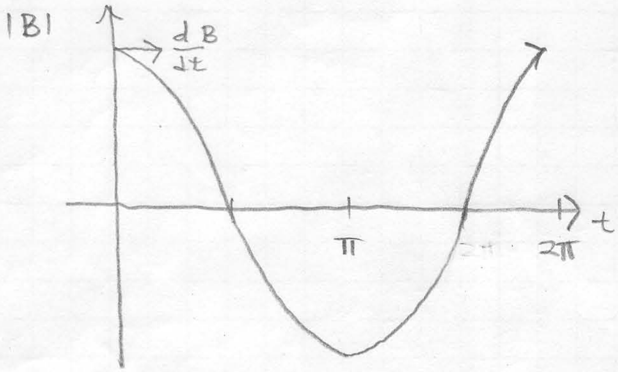
b) Determine the direction and magnitude of the current that would flow through a $4\,\Omega$ resistor connected across the gap.

Assume the internal resistance of the loop is $1\,\Omega$,

$$I = \frac{V_{emf}}{4+1} \qquad (6.9)$$



$$= \frac{\mu_0 I_0 \cdot 0.1}{2\pi} \sin(2\pi \cdot 10^4 t)$$



$$\bar{B} = \hat{\phi} B_\phi \cos(\omega t) \quad \frac{Wb}{m}$$

Lenz's Low

Polarity of V^{tr}_{emf} and hence direction of current in loop is always in opposition to the change of magnetic flux $\Phi(t)$ that produces I.