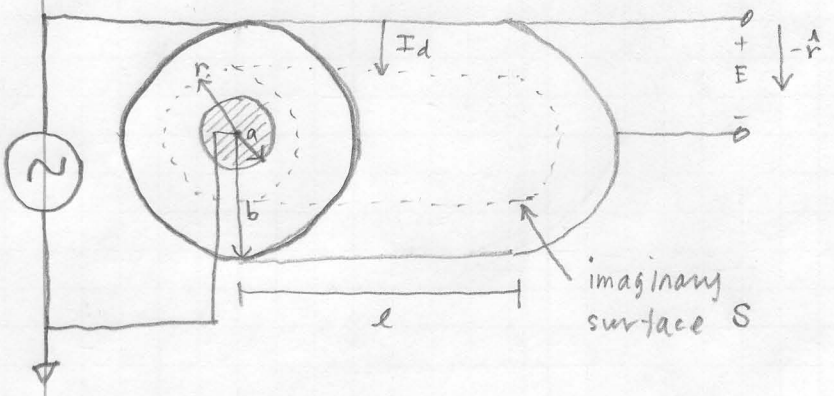
6.15 A coaxial capacitor of length $l = 6$ cm uses an insulating dielectric material $\varepsilon_r = 9$. The radii of the cylindrical conductors are 0.5 cm and 1 cm.

Determine the displacement current if $V(t) = 50 \sin 120\pi t$ (V).



$$\bar{E} = -\hat{r}\frac{Q}{2\pi\varepsilon r l} \qquad (4.114)$$

$$V = \frac{Q}{2\pi\varepsilon l}\ln\left(\frac{b}{a}\right) \qquad (4.115)$$

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$$Q = \frac{V\,2\pi\varepsilon l}{\ln(b/a)}, \qquad \tilde{V} = 50\,e^{-j\frac{\pi}{2}} = -j50 \quad (V)$$

$$\bar{E} = -\hat{r}\frac{V\,2\pi\varepsilon l}{\ln\left(\frac{b}{a}\right)2\pi\varepsilon r l} = -\hat{r}\frac{V}{\ln(b/a)\,r} = \mathrm{Re}\left\{\tilde{E}(r)\,e^{-j\frac{\pi}{2}}\right\} \quad (V/m)$$

$$\tilde{E}(r) = -\hat{r}\frac{\tilde{V}}{\ln(b/a)}\frac{1}{r} = \hat{r}\frac{jV_0}{\ln(b/a)}\frac{1}{r} = \hat{r}\frac{1}{r}jE_0$$

$$\tilde{D}(r) = \varepsilon\tilde{E}(r) = \hat{r}\frac{\varepsilon}{r}jE_0$$

$$D(r) = \mathrm{Re}\left\{\varepsilon\tilde{E}(r)\right\} = -\hat{r}\frac{\varepsilon V_0\sin(\omega t)}{\ln(b/a)}\frac{1}{r} \quad (C/m^2)$$

$$\bar{S} = -\hat{r}\,2\pi r l$$

$$I_d = \frac{\partial}{\partial t}\bar{D}\cdot\bar{S} = -\hat{r}\frac{\varepsilon V_0\cos(\omega t)}{\ln(b/a)}\frac{1}{r}\cdot -\hat{r}\,2\pi r l$$

$$= \frac{\varepsilon V_0\cos(\omega t)\,2\pi l\,\omega}{\ln(b/a)}$$

$$= \frac{9\cdot 8.85\times10^{-12}\cdot 2\pi^2\cdot 0.06\cdot 120}{\ln\left(\frac{1}{0.5}\right)}\cos(120\pi t) = \boxed{816.6\cos(120\pi t)\ n(A/m)}$$