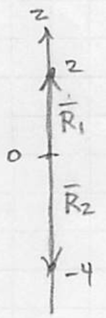
4.38 Given $\bar{E} = \hat{R}\,\frac{18}{R^2}$ V/m determine the electric potential of point A with respect to point B.

$A = (0,0,2)$, $B = (0,0,-4)$



$$V_{AB} = V_A - V_B = -\int_B^A \bar{E}\cdot d\bar{l}$$

$$= -\int_{-4}^{2} \bar{E}\cdot d\bar{l}$$

note: discontinuity at $z=0$

$\bar{R}_1 = \hat{z}\,\frac{18}{z^2} \quad 0 \le z \le 2$

$\bar{R}_2 = -\hat{z}\,\frac{18}{z^2} \quad -4 \le z \le 0$

$$= -\left(\int_{-4}^{0} \bar{R}_2\cdot d\bar{l} + \int_0^2 \bar{R}_1\cdot d\bar{l}\right)$$

$$= -\left(\int_{-4}^{0}\left(-\hat{z}\,\frac{18}{z^2}\right)\cdot\hat{z}\,dz + \int_0^2 \hat{z}\,\frac{18}{z^2}\cdot\hat{z}\,dz\right)$$

$$= -\left(\int_{-4}^{0}\frac{18}{z^2}\,dz + \int_0^2 \frac{18}{z^2}\,dz\right)$$

These are improper integrals so limits must be taken

$$= -\left(\lim_{t\to\infty}\left(\int_{-4}^{t}\frac{18}{z^2}\,dz + \int_t^2\frac{18}{z^2}\,dz\right)\right)$$

$$= 18\left(\lim_{t\to\infty}\left(\frac{1}{t}+\frac{1}{4}\right) - \lim_{t\to\infty}\left(\frac{1}{2}-\frac{1}{t}\right)\right)$$

$$\boxed{= 4.5\ \text{V}}$$