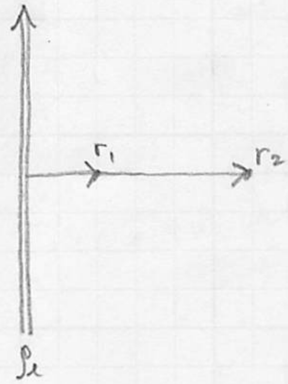
4.33 Show that the electric potential difference V_{12} between two points in air at radial distance r_1 and r_2 from an infinite line of charge with density ρ_ℓ along the z-axis is $V_{12} = (\rho_\ell / 2\pi\varepsilon_0)\ln(r_2/r_1)$.



$$\bar{E} = \hat{r}\frac{\rho_\ell}{2\pi\varepsilon r} \qquad \text{EQ } (4.33)$$

$$V_{12} = V_1 - V_2 = -\int_{r_2}^{r_1} \bar{E}\cdot d\bar{l}$$

$$= -\int_{r_2}^{r_1} \hat{r}\frac{\rho_\ell}{2\pi\varepsilon r}\cdot \hat{r}\,dr$$

$$= -\int_{r_2}^{r_1} \frac{\rho_\ell}{2\pi\varepsilon r}\,dr$$

$$= \frac{-\rho_\ell}{2\pi\varepsilon}\int_{r_2}^{r_1}\frac{1}{r}\,dr$$

$$= \frac{-\rho_\ell}{2\pi\varepsilon}\left(\ln(r)\right)\Big|_{r_2}^{r_1}$$

$$= \frac{-\rho_\ell}{2\pi\varepsilon}\left(\ln(r_1) - \ln(r_2)\right)$$

$$= \frac{\rho_\ell}{2\pi\varepsilon}\left(\ln(r_2) - \ln(r_1)\right)$$

$$= \frac{\rho_\ell}{2\pi\varepsilon}\ln(r_2/r_1) \quad \checkmark$$

Note: in air $\varepsilon_r = 1$ ∴ $\varepsilon = \varepsilon_0$