



$$\vec{E} = \hat{x} xz - \hat{y} yz^2 - \hat{z} xy$$

$$\oint_S \vec{E} \cdot d\vec{s} = F_{\text{top}} + F_{\text{bottom}} + F_{\text{left}} + F_{\text{right}} + F_{\text{front}} + F_{\text{back}}$$

$$F_{\text{front}} = \int_{-1}^1 \int_{-1}^1 \vec{E} \cdot \hat{n}_{\text{front}} ds = \int_{-1}^1 \int_{-1}^1 (\hat{x} xz - \hat{y} yz^2 - \hat{z} xy) \Big|_{z=1} \cdot \hat{z} dx dy$$

↖ front plane

$$= \int_{-1}^1 \int_{-1}^1 (-xy) dx dy$$

$$= - \int_{-1}^1 \int_{-1}^1 xy dx dy$$

$$= - \int_{-1}^1 \left. \frac{x^2}{2} \right|_{-1}^1 y dy$$

$$= - \int_{-1}^1 \left(\frac{1}{2} - \frac{1}{2} \right) y dy$$

$$= 0$$

F_{back} will also be 0

$$F_{\text{left}} = \int_{-1}^1 \int_{-1}^1 (\hat{x} xz - \hat{y} yz^2 - \hat{z} xy) \Big|_{x=-1} \cdot (-\hat{x}) dz dy$$

$$= \int_{-1}^1 \int_{-1}^1 (-1)z dz dy$$

$$= 0$$

F_{right} will also be 0

make sure on the homework you show that you know how to setup these equations. i.e. F_{back} & F_{right}

$$F_{\text{top}} = \int_{-1}^1 \int_{-1}^1 (x^1 xz - y^1 yz^2 - z^1 xy) \Big|_{y=1} \cdot \hat{y} dz dx$$

$$= \int_{-1}^1 \int_{-1}^1 (-z^2) dz dx$$

$$= x \left[-\frac{z^3}{3} \right]_{-1}^1$$

$$= -2 \left(\frac{1}{3} + \frac{1}{3} \right)$$

$$= -\frac{4}{3}$$

$$F_{\text{bottom}} = \int_{-1}^1 \int_{-1}^1 (x^1 xz - y^1 yz^2 - z^1 xy) \Big|_{y=-1} \cdot (-\hat{y}) dz dx$$

$$= \int_{-1}^1 \int_{-1}^1 -z^2 dz dx$$

$$= -\frac{4}{3}$$

$$\oint \vec{E} \cdot d\vec{s} = F_{\text{top}} + F_{\text{bottom}} + F_{\text{left}} + F_{\text{right}} + F_{\text{front}} + F_{\text{back}}$$

$$= -\frac{4}{3} - \frac{4}{3} + 0 + 0 + 0 + 0$$

$$= -\frac{8}{3}$$