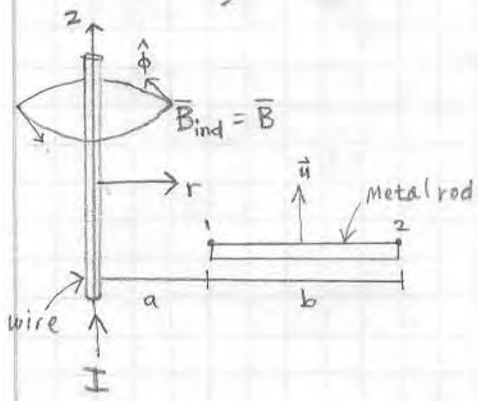
ECE332 Example 6-5 Ulaby

Rod moving Next to a Wire



An "infinite" length of wire in freespace carriers a current $I = 10A$. A 0.3m metal rod is 0.1m away from it and moves at a constanty velocity $\vec{u} = \hat{z}\,5\ m/s$.

Determine V_{12}.

The current I in the wire induces a magnetic field, using Ampere's Law.

$$\nabla \times \bar{H} = \bar{J}$$

Note:

strokes theorem $\int_S \nabla \times \bar{F} \cdot d\bar{s} = \oint_C \bar{F} \cdot d\bar{l}$ (1.107)

$I = \int_S \bar{J} \cdot d\bar{s}$ (4.12) and $\bar{B} = \mu \bar{H}$ (5.2)

$$\int_S (\nabla \times \bar{H}) \cdot d\bar{s} = \int_S \bar{J} \cdot d\bar{s}$$

$$\oint_C \frac{\bar{B}}{\mu} \cdot d\bar{l} = I$$

To satisfy the R.H.R, since I is in the $+\hat{z}$ direction $\bar{H}$ must be in the $+\hat{\phi}$ direction.

$$\int_0^{2\pi} \hat{\phi} \frac{B_\phi}{\mu} \cdot \hat{\phi}\, r\, d\phi = I$$

differential length, $dl_\phi = \hat{\phi}\, r\, d\phi$

$$\int_0^{2\pi} \frac{B_\phi}{\mu} r\, d\phi = I$$

Note: $\hat{\phi} \cdot \hat{\phi} = 1$

$$2\pi \frac{B_\phi}{\mu} r = I$$

$$B_\phi = \frac{\mu I}{2\pi r}$$

$$\boxed{\bar{B} = \hat{\phi} \frac{\mu I}{2\pi r}}$$

Note: magnetic field is in freespace so $\mu_r = 1$ and $\mu = \mu_r \mu_0 = \mu_0$.

Since the magnetic field is static (time-independent) as the conducting metal rod moves through the field the free electrons in the rod experience the influence of a Magnetic Force F_m

$$F_m = q\,\vec{u} \times \bar{B} \quad (5.3)$$

This magnetic force is equivalent to the electrical force that would be exerted on the particle by an electric field $\bar{E}_m$.

$$\bar{E}_m = \frac{\bar{F}_m}{q} = \vec{u} \times \bar{B} \qquad (6.23)$$

The field $\bar{E}_m$ generated by the motion of the charged particle is called the motional electric field and is in the direction perpendicular to the plane containing $\vec{u}$ and $\bar{B}$.

The magnetic force F_m acting on the electrons in the metal rod cause them to move in the direction of $\bar{E}_m$. This induces a potential difference across the rod. The voltage difference across the rod is called the motional emf V^m_{emf}.

$$V_{12} = V_{emf} = \cancelto{0}{V^{tr}_{emf}} + V^m_{emf} = V^m_{emf}$$

Note: static magnetic field so $V^{tr}_{emf} = 0$.

$$V^m_{emf} = \int_2^1 (\vec{u} \times \bar{B}) \cdot d\bar{l}$$

$$= \int_{(a+b)}^{a} (\hat{z}5 \times \hat{\phi} B_\phi) \cdot \hat{r}\, dr$$

$$\hat{z}5 \times \hat{\phi} B_\phi = \begin{vmatrix} \hat{r} & \hat{\phi} & \hat{z} \\ 0 & 0 & 5 \\ 0 & B_\phi & 0 \end{vmatrix} \begin{matrix} \hat{r} & \hat{\phi} \\ 0 & 0 \\ 0 & B_\phi \end{matrix}$$

$$= -\hat{r}\, 5 B_\phi$$

$$= \int_{(a+b)}^{a} \hat{r}(-5 B_\phi) \cdot \hat{r}\, dr$$

Note: $\hat{r} \cdot \hat{r} = 1$

$$= \int_{(a+b)}^{a} (-5 B_\phi)\, dr$$

$$= \int_{(a+b)}^{a} \left(-5 \frac{\mu I}{2\pi r}\right) dr$$

$$= \frac{-5\mu I}{2\pi} \int_{(a+b)}^{a} r^{-1}\, dr$$

$$= \frac{-5\mu I}{2\pi} \left(\ln(r)\right)\Big|_{(a+b)}^{a}$$

$$= \frac{-5\mu I}{2\pi} \left(\ln(a) - \ln(a+b)\right)$$

$$\boxed{V^m_{emf} = \frac{-5\mu I}{2\pi} \ln\left(\frac{a}{a+b}\right)}$$

$$V_{12} = V^m_{emf} = -\frac{5 \cdot 4\pi \times 10^{-7} \cdot 10}{2\pi} \ln\left(\frac{0.1}{0.4}\right)$$

$$= -100 \times 10^{-7} \ln(0.25)$$

$$= 13.86\ \mu V$$