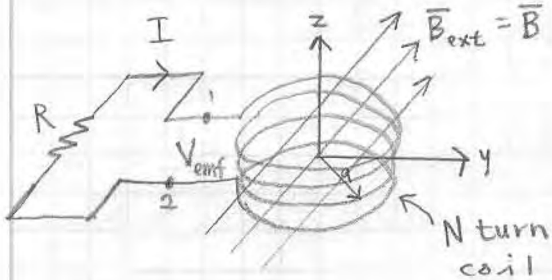
Inductor in a changing Magnetic Field



An inductor is formed by winding N turns of a conducting wire into circular loops of radius a. The inductor loop is centered at the origin in the x-y plane and connected to a resistor R.

In the presence of an external magnetic field $\bar{B}_{ext} = B_0(\hat{y}2 + \hat{z}3)\sin(wt)$

Note: The external magnetic field is time varying therefore according to Faraday's law (6.6) an electromotive force is induced.

$$V_{emf} = V_{emf}^{tr} + V_{emf}^{m} = V_{emf}^{tr} + 0$$

Since the loop is stationary the motional emf equals 0 (p. 257).

The magnetic flux density linking each coil is given by:

$$\Phi = \int_S \bar{B} \cdot d\bar{s} \quad (6.5)$$

Note: The surface we are integrating over is in the x-y plane, therefore $\hat{z} = \hat{n}$ and we need to use $ds_z = \hat{z}\, r\, dr\, d\theta$ (Table 3-1) cylindrical coordinates used to match problem geometry.

$$\Phi = \int_0^{2\pi} \int_0^a \left(B_0(\hat{y}2 + \hat{z}3)\sin(wt)\right) \cdot \hat{z}\, r\, dr\, d\theta$$

$$= \int_0^{2\pi} \int_0^a \left(\hat{y} B_0 2\sin(wt) + \hat{z} B_0 3\sin(wt)\right) \cdot \hat{z}\, r\, dr\, d\theta$$

Note: $\hat{z} \cdot \hat{y} = 0$

$\hat{z} \cdot \hat{z} = 1$

$$= \int_0^{2\pi} \int_0^a B_0 3\sin(wt)\, r\, dr\, d\theta$$

$$= 3B_0 \sin(wt) \int_0^{2\pi} d\theta \int_0^a r\, dr$$

$$= 3B_0 \sin(wt)(\cancel{2}\pi)\left(\frac{r^2}{\cancel{2}}\right)\Big|_0^a$$

$$\boxed{\Phi = 3\pi a^2 B_0 \sin(wt) \quad \text{Wb}}$$

Note: If the surface area is planar and the magnetic field is uniform then to save time we should realize that the Φ is just $B_{\hat{n}} A_s$ where A_s is the surface area.

$$A_s = \pi a^2, \quad B_{\hat{n}} = B_z = B_0 3\sin(wt)$$

$$\Phi = B_z \cdot A_s = 3\pi a^2 B_0 \sin(wt)$$

if $N=10$, $B_0 = 0.2\ T$, $a = 0.1\ m$, $w = 10^3\ rad/s$

$$V_{emf} = V_{emf}^{tr} = -N\frac{d\Phi}{dt} = -N\frac{d}{dt}\int_S \bar{B}\cdot d\bar{s} = -N\frac{d}{dt}\left(3\pi a^2 B_0 \sin(wt)\right)$$

$$= -N\, 3\pi a^2 B_0 \cos(wt)\, w$$

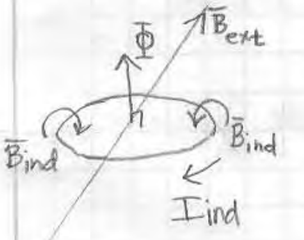
$$= -10\cdot 3\pi \cdot 0.1^2 \cdot 0.2 \cos\left(10^3 t\right) 10^3$$

$$\boxed{V_{emf} = -60\pi \cos\left(10^3 t\right) \quad V}$$

At $t=0$ $\quad \frac{d\Phi}{dt} = \frac{d}{dt}\left(3\pi a^2 B_0 \sin(wt)\right) = 3\pi a^2 B_0\, w \cos(wt) = 3\pi a^2 B_0 w$

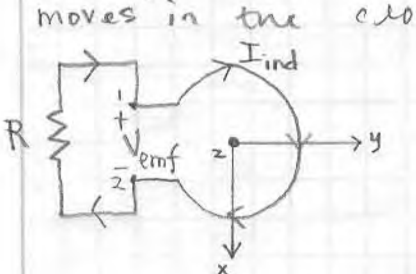
Since $\frac{d\Phi}{dt} > 0$ the external $\bar{B}(t)$ is increasing and from ...

Lenz's Law; the current in the loop is always in such a direction as to oppose the change of magnetic flux $\Phi(t)$ that produced it.



from the R.H.R. for magnetic field near a wire, thumb in direction of current with four fingers curled around the wire in the direction of the magnetic field.

Looking down on the circuit from $+\hat{z}$ the induced current moves in the clockwise direction.



Since the current propagates from node 1 to node 2, node 1 must be at a higher potential (positive).

$$V_{emf} = V_{emf}^{tr} = V_1 - V_2 = -60\pi \approx -188.5\ V \ @ t=0$$

For $R = 1\,k\Omega$ $\quad V_{emf} = I_{ind} R$

(neglecting wire resistance)

$$\boxed{I_{ind}(t) = \frac{V_{emf}(t)}{R} = \frac{-60\pi\cos\left(10^3 t\right)}{R}}$$

At $t=0$ $\quad I_{ind}(0) = \frac{-60\pi\cos(0)}{10^3} \approx -188.5\ mA$