Example 3-10

$$\nabla T = \text{grad}\, T \triangleq \hat{x}\frac{\partial T}{\partial x} + \hat{y}\frac{\partial T}{\partial y} + \hat{z}\frac{\partial T}{\partial z} \qquad (3.72)$$

$$\text{cylindrical } \nabla = \hat{r}\frac{\partial}{\partial r} + \hat{\phi}\frac{1}{r}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z} \qquad (3.82)$$

$$\text{spherical } \nabla = \hat{R}\frac{\partial}{\partial R} + \hat{\theta}\frac{1}{R}\frac{\partial}{\partial \theta} + \hat{\phi}\frac{1}{R\sin\theta}\frac{\partial}{\partial \phi} \qquad (3.83)$$

$$V_1 = 24\, V_0 \cos\left(\frac{y}{3}\pi\right)\sin\left(\frac{2\pi}{3}z\right)$$

Note: gradient operator results in a vector field. Take the partial derivative with respect to the corresponding unit vector.

$$\nabla V_1 = \hat{x}\frac{\partial V_1}{\partial x} + \hat{y}\frac{\partial V_1}{\partial y} + \hat{z}\frac{\partial V_1}{\partial z}$$

The cylindrical and spherical have some extras included to account for the coordinate conversions. Check the back cover of Ulaby for all the necessary equations.

$$= \hat{x}\frac{\partial}{\partial x}\left(24\, V_0 \cos\left(\frac{y}{3}\pi\right)\sin\left(\frac{2\pi}{3}z\right)\right) +$$

$$\hat{y}\frac{\partial}{\partial y}\left(24\, V_0 \cos\left(\frac{y}{3}\pi\right)\sin\left(\frac{2\pi}{3}z\right)\right) +$$

$$\hat{y}\frac{\partial}{\partial z}\left(24\, V_0 \cos\left(\frac{y}{3}\pi\right)\sin\left(\frac{2\pi}{3}z\right)\right)$$

$$= \hat{x}(0) + \hat{y}\, 24\, V_0 \sin\left(\frac{2\pi}{3}z\right)\frac{\partial}{\partial y}\cos\left(\frac{y}{3}\pi\right) + \hat{z}\, 24\, V_0 \cos\left(\frac{y}{3}\pi\right)\frac{\partial}{\partial z}\sin\left(\frac{2\pi}{3}z\right)$$

$$= \hat{y}\, 24\, V_0 \sin\left(\frac{2\pi}{3}z\right)\left(-\frac{\pi}{3}\sin\left(\frac{y}{3}\pi\right)\right) + \hat{z}\, 24\, V_0 \cos\left(\frac{y}{3}\pi\right)\left(\frac{2\pi}{3}\cos\left(\frac{2\pi}{3}z\right)\right)$$

$$= \hat{y}\,\frac{-24\pi}{3} V_0 \sin\left(\frac{2\pi}{3}z\right)\sin\left(\frac{y}{3}\pi\right) + \hat{z}\,\frac{48\pi}{3} V_0 \cos\left(\frac{y}{3}\pi\right)\cos\left(\frac{2\pi}{3}z\right)$$

$$= -\hat{y}\, 8\pi V_0 \sin\left(\frac{2\pi}{3}z\right)\sin\left(\frac{y}{3}\pi\right) + \hat{z}\, 16\, V_0 \cos\left(\frac{y}{3}\pi\right)\cos\left(\frac{2\pi}{3}z\right)$$