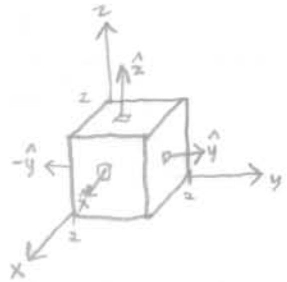
a) $\oint_s \bar{E} \cdot d\bar{s} = F_{top} + F_{bottom} + F_{left} + F_{right} + F_{front} + F_{back}$



$$F = \int_s \bar{E} \cdot \hat{n}\, d\bar{s}$$

$$F_{top} = \int_0^2 \int_0^2 \bar{E} \cdot \hat{z}\, dx\, dy$$

Base vector properties

$\hat{x} \cdot \hat{x} = 1$
$\hat{x} \cdot \hat{z} = 0$
$\hat{x} \cdot \hat{y} = 0$

$$= \int_0^2 \int_0^2 \hat{x} xz - \hat{y} yz^2 - \hat{z} xy \Big|_{z=2} \cdot \hat{z}\, dx\, dy$$

$$= \int_0^2 \int_0^2 \hat{x}\, 2x - \hat{y}\, 4y - \hat{z}\, xy \cdot \hat{z}\, dx\, dy$$

$$= \int_0^2 \int_0^2 (-xy)\, dx\, dy$$

$$= -\left[\frac{x^2 y^2}{4} \Big|_0^2 \Big|_0^2 \right]$$

$$= -\left[\frac{4}{4} y^2 \Big|_0^2 \right]$$

$$\underline{\underline{= -4}}$$ (repeat this process for each side, ulaby pg. 134)

b) $\oint_v \nabla \cdot \bar{E}\, dv = \int_0^2 \int_0^2 \int_0^2 \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}\, dx\, dy\, dz$

$$= \int_0^2 \int_0^2 \int_0^2 \frac{\partial}{\partial x}(xz) + \frac{\partial}{\partial y}(-yz^2) + \frac{\partial}{\partial z}(-xy)\, dx\, dy\, dz$$

$$= \int_0^2 \int_0^2 \int_0^2 (z + (-2z) + 0)\, dx\, dy\, dz$$

$$= \int_0^2 \int_0^2 \int_0^2 (z - 2z)\, dz\, dx\, dy = \underline{\underline{-8/3}}$$

you should get $-8/3$ for both parts $\therefore$ divergence theorem is verified.