1. Write **countable** or **uncountable** to indicate the cardinality of each set.
   a. Rational numbers ________________________________
   b. Positive real numbers ________________________________
   c. \( \mathbb{N} \cup \mathbb{N} \) ________________________________
   d. \( \text{power}(\mathbb{N}) \) ________________________________
   e. \( \mathbb{N} \times \mathbb{N} \times \mathbb{N} \) ________________________________
   f. \( \{a, b\}^* \) ________________________________

2. Write an inductive definition for each set.
   a. \( S = \{a\}^* \times \{b\}^* \). Assume that the basis case is: \((\Lambda, \Lambda) \in S\).

   b. \( S = \{<1>, <3, 1>, <5, 3, 1>, <7, 5, 3, 1>, \ldots\} \).

3. Show each step in the calculation of \( f(47) \), where \( f \) is defined by
   \[
   f(0) = 0 \\
   f(n) = f(\text{floor}(n/3)) + n
   \]

4. Write a recursive definition for the following function.
   \( f(n) = 4 + 6 + \ldots + (2n + 4) \), where \( n \in \mathbb{N} \).
6. Write a recursive definition for the procedure leaves, where for a binary tree $T$, let $leaves(T)$ be a procedure to print out the leaves of $T$ as they occur from left to right.

7. For each of the following relations, write down the properties that the relation satisfies from the list: reflexive, symmetric, transitive, irreflexive, antisymmetric.
   a. isParentOf, over the set of people.
   b. $\neq$, over the set $\mathbb{N}$ of natural numbers.
   c. isSubsetOf, over a collection of sets.

8. Given the following binary relations over $\{a, b, c, d\}$.
   $\begin{align*}
   R &= \{(a, b), (b, c), (c, c), (d, c)\} \\
   S &= \{(b, a), (c, b), (c, d)\}
   \end{align*}$
   a. Find $R \circ S$
   b. Find $S \circ R$

9. Find the transitive closure of $R = \{(1, 2), (3, 1), (3, 2), (2, 4)\}$.
10. Given the following weighted graph.

a. Draw a matrix that can be used to look up the length of shortest paths between any two points.

b. Draw a path matrix that can be used to compute the shortest path between any two points.