Some Important Proofs on Sequence Pair Representations of Floorplans

Given: A sequence-pair $\{\Gamma^+, \Gamma^-\}$ for K rectangular modules

Def: (*cluster*)

A set of proximate modules whose topology remains fixed over all possible floorplans.

Def: (*cluster sequence*)

Let $\mathcal{C}$ be a cluster. The cluster sequence is the minimum length subsequence in a sequence-pair that contains all modules $r \in \mathcal{C}$.

Def: (*cluster gap*)

Any position(s) in a cluster sequence that contains a module $y \notin \mathcal{C}$.

Def: (*cluster gap module*)

Any module located within a cluster gap.

For any cluster $\mathcal{C}$, there is a cluster sequence $\mathcal{S}_\mathcal{C}^+$ contained in Γ^+ and another cluster sequence $\mathcal{S}_\mathcal{C}^-$ contained in Γ^-. We indicate an ordering within a cluster sequence by $x \prec y$, which says x appears before y when the cluster sequence is scanned from left to right. In general $\mathcal{S}_\mathcal{C}^+ \neq \mathcal{S}_\mathcal{C}^-$ because each may contain different cluster gap modules. In fact, even if $\mathcal{S}_\mathcal{C}^+$ and $\mathcal{S}_\mathcal{C}^-$ contain the exact same modules, the ordering will most likely be different.

Consider a $m \times m$ mesh with the horizontal and vertical mesh lines labeled by Γ^+ and Γ^-. (The labeling is done top-to-bottom and left-to-right.) Figure 1 is the mesh constructed for the sequence-pair $\Gamma^+ = \{f, c, b, g, e, a, d\}$ and $\Gamma^- = \{g, b, e, c, d, a, f\}$. The cluster consists of modules a, b, c and e—i.e., $\mathcal{C} = \{a, b, c, e\}$. Consequently, module g is a cluster gap module in Γ^+, whereas module d is a cluster gap module in Γ^-. The module x is placed at the crosspoint (x, x).

Def: (*crosspoint*)

Any point in a mesh with coordinates (r, r), where $r \in \Gamma^+$ and $r \in \Gamma^-$.

Def: (submesh)

Let l^+ and r^+ be the leftmost and rightmost modules respectively in $\mathcal{S}_\mathcal{C}^+ \subset \Gamma^+$ and l^- and r^- the same for $\mathcal{S}_\mathcal{C}^- \subset \Gamma^-$. The submesh is bounded by the horizontal and vertical lines of the mesh with labels l^+, r^+, l^- and r^-.

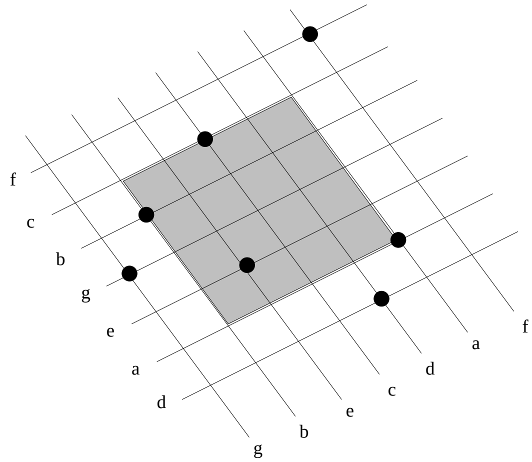

Figure 1: A mesh constructed for the sequence-pair $\Gamma^+ = \{f, c, b, g, e, a, d\}$ and $\Gamma^- = \{g, b, e, c, d, a, f\}$. The cluster is $\mathcal{C} = \{a, b, c\}$ and the corresponding submesh is shaded. The crosspoints are shown as black dots.

Theorem 1 *A submesh interior only contains crosspoints of modules that are in both $\mathcal{S}_\mathcal{C}^+$ and $\mathcal{S}_\mathcal{C}^-$.*

Proof. (by contradiction) Suppose the crosspoint of y lies in the submesh interior of a cluster $\mathcal{C}$, but assume y is a module in either $\mathcal{S}_\mathcal{C}^+$ or $\mathcal{S}_\mathcal{C}^-$ (but not both).

The boundary of the submesh is defined by the leftmost module l and the rightmost module r of a cluster sequence. Hence, any module y is to the right of l and to the left of r in the cluster sequence—i.e., $l \prec y \prec r$.

Without loss in generality, assume y is in $\mathcal{S}_\mathcal{C}^+$ only, which means only the mesh rows need to be considered. By following the mesh labeling procedure, there exists a horizontal row with a y label between the rows with labels l^+ and r^+—i.e., $l^+ \prec y \prec r^+$.

Now assume y is in $\mathcal{S}_\mathcal{C}^-$ only. By the same reason stated above, there exists a column with a y label between the columns with labels l^- and r^-.

Note that the crosspoint of y lies in the submesh interior iff $l^+ \prec y \prec r^+$ and $l^- \prec y \prec r^-$ both hold. But this means y must be in both $\mathcal{S}_\mathcal{C}^+$ and $\mathcal{S}_\mathcal{C}^-$, which contradicts the initial assumption. □

Corollary 1 *The crosspoint of any module $a_i \in \mathcal{C}$ lies on the boundary or in the interior of the submesh associated with $\mathcal{C}$.*

Proof. Clearly a_i is on the boundary whenever $a_i = l^+, l^-, r^+$ or r^-. The proof for the interior follows from Theorem 1 and the definition of a cluster sequence. □

Referring again to Figure 1, module e is a cluster gap module in both Γ^+ and Γ^- and its crosspoint is in the submesh interior. On the other hand, modules g and d are cluster gap modules in only one sequence-pair while f is not a cluster module at all; none of these modules are in the submesh interior.

We adopt the sequence-pair notation used in Murata's paper[1]:

- $\mathcal{M}^{bb}(x) = \{x' | x' \text{ is before } x \text{ in both } \Gamma^+ \text{ and } \Gamma^-\}$
- $\mathcal{M}^{ba}(x) = \{x' | x' \text{ is before } x \text{ in } \Gamma^+ \text{ and after } x \text{ in } \Gamma^-\}$
- $\mathcal{M}^{ab}(x) = \{x' | x' \text{ is after } x \text{ in } \Gamma^+ \text{ and before } x \text{ in } \Gamma^-\}$
- $\mathcal{M}^{aa}(x) = \{x' | x' \text{ is after } x \text{ in } \Gamma^+ \text{ and after } x \text{ in } \Gamma^-\}$

These two sets are used to construct the horizontal-constraint digraph (G_H) and the vertical-constraint digraph (G_V), from which the overall chip area can be computed.

Theorem 2 *If the ordering between modules a_i and a_j given by $\prec$ holds over all $\mathcal{S}_\mathcal{C}^+$ and $\mathcal{S}_\mathcal{C}^-$, then a_i and a_j are modules in $\mathcal{C}$.*

Proof. By the definition of a cluster sequence, a_i and a_j must appear in all $\mathcal{S}_\mathcal{C}^+$ and $\mathcal{S}_\mathcal{C}^-$. Suppose $a_i \prec a_j$ in all $\mathcal{S}_\mathcal{C}^+$ and $\mathcal{S}_\mathcal{C}^-$. Then in all cluster sequences we have $a_i \in \mathcal{M}^{bb}(a_j)$, which means a_i must always be to the left of a_j. Similarly, if $a_i \prec a_j$ in all $\mathcal{S}_\mathcal{C}^+$, but $a_j \prec a_i$ in all $\mathcal{S}_\mathcal{C}^-$, then $a_j \in \mathcal{M}^{ab}(a_i)$ in all cluster sequences, which means a_j must always be above a_i. Either situation establishes a fixed topology of between modules a_i and a_j over all floorplans. □

Theorem 3 *Let $\{\widehat{\Gamma^+}, \widehat{\Gamma^-}\}$ be any permutation of $\{\Gamma^+, \Gamma^-\}$ and let a_i and a_j be two arbitrary modules in a cluster $\mathcal{C}$. If the ordering $\prec$ between a_i and a_j is identical in both $\{\widehat{\Gamma^+}, \widehat{\Gamma^-}\}$ and $\{\Gamma^+, \Gamma^-\}$, then $\mathcal{C}$ exists in the floorplans defined by $\{\widehat{\Gamma^+}, \widehat{\Gamma^-}\}$ and $\{\Gamma^+, \Gamma^-\}$.*

Proof. It is always possible to construct a $\mathcal{S}_\mathcal{C}^+ \subseteq \Gamma^+$ and a $\mathcal{S}_\mathcal{C}^- \subseteq \Gamma^-$. Let $\{\widehat{\mathcal{S}_\mathcal{C}^+}, \widehat{\mathcal{S}_\mathcal{C}^-}\}$ denote the cluster sequence-pairs of $\{\widehat{\Gamma^+}, \widehat{\Gamma^-}\}$. If the ordering of a_i and a_j in $\{\widehat{\Gamma^+}, \widehat{\Gamma^-}\}$ and $\{\Gamma^+, \Gamma^-\}$ is identical, then the ordering is also identical in $\{\widehat{\mathcal{S}_\mathcal{C}^+}, \widehat{\mathcal{S}_\mathcal{C}^-}\}$ and $\{\mathcal{S}_\mathcal{C}^+, \mathcal{S}_\mathcal{C}^-\}$. This will be true in any permutation of $\{\Gamma^+, \Gamma^-\}$ where ordering of a_i and a_j is preserved. Consequently, it is true in all such permutations and therefore all $\{\widehat{\mathcal{S}_\mathcal{C}^+}, \widehat{\mathcal{S}_\mathcal{C}^-}\}$. By Theorem 2, a_i and a_j are in the same cluster $\mathcal{C}$. □

Corollary 2 *Given a module $y \notin \mathcal{C}$. Then any permutation of Γ^+ or Γ^- that only alters the position of y will preserve $\mathcal{C}$.*

Proof. By only altering the position of y, no other ordering of other modules in Γ^+ or Γ^- is affected. By Theorem 3 the cluster remains intact. □

[1] H. Murata et. al, "VLSI module placement based on rectangle-packing by the sequence-pair", *IEEE Trans. Comput. -Aided Des.* 15(12), 1518-1524, 1996

We now consider the following issue. The ordering talked about up to this point only establishes a relative positioning between modules in a cluster—i.e., any pretense of physical distances has been ignored. Figure 2 shows the real issue. Our objective now is to form clusters that looks like Figure 2(b). What we want is *compact clusters.*

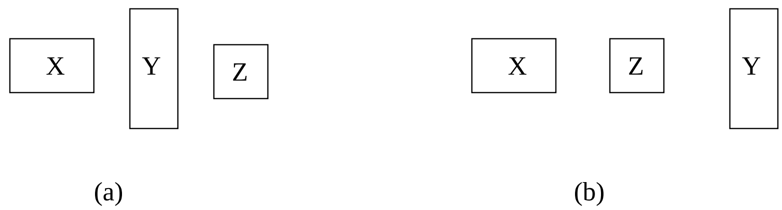

Figure 2: Two possible positions of cluster modules x and z. In (a), a cluster gap module y is in between the cluster modules, while in (b) the cluster modules are next to each other. However, in both cases the cluster position completely complies with $x \in \mathcal{M}^{bb}(z)$.

Def: (*compact cluster*)

> A cluster $\mathcal{C}$ is called compact if $\forall$ modules $x, z \in \mathcal{C}$, there exists no module $y \notin \mathcal{C}$ which lies within any portion of the deadspace between x and z.

Theorem 4 *Let $\mathcal{C}$ be a compact cluster. Then the same cluster gap module cannot appear in both $\mathcal{S}_{\mathcal{C}}^{+}$ and $\mathcal{S}_{\mathcal{C}}^{-}$.*

Proof. (by contradiction) Assume $\mathcal{C}$ is a compact cluster. Let y be a cluster gap module—i.e., $y \notin \mathcal{C}$—but $y \in \mathcal{S}_{\mathcal{C}}^{+}$ and $y \in \mathcal{S}_{\mathcal{C}}^{-}$.

If y is the first or the last module in $\mathcal{S}_{\mathcal{C}}^{+}$ or $\mathcal{S}_{\mathcal{C}}^{-}$, then by the definition of a cluster sequence, we must have $y \in \mathcal{C}$. Therefore, we only consider the case where y is not the first or last module in either cluster sequence.

Without loss in generality, let l be the leftmost and r the rightmost module in $\mathcal{S}_{\mathcal{C}}^{+}$ and $\mathcal{S}_{\mathcal{C}}^{-}$. Then in both pairs $l \prec y$, or $l \in \mathcal{M}^{bb}(y)$ and $y \prec r$, or $y \in \mathcal{M}^{bb}(r)$. This means y is to the right of l but to the left of r—i.e., y is in between l and r, which contradicts the original assumption that $\mathcal{C}$ is compact.

Now, let $y \in \mathcal{S}_{\mathcal{C}}^{+} \Rightarrow y \notin \mathcal{S}_{\mathcal{C}}^{-}$. Then only one of the following cases exists:

1. $y \in \mathcal{M}^{aa}(l)$ and $y \in \mathcal{M}^{ba}(r)$
2. $y \in \mathcal{M}^{ab}(l)$ and $y \in \mathcal{M}^{bb}(r)$

Since $l \prec r$ in both $\mathcal{S}_{\mathcal{C}}^{+}$ and $\mathcal{S}_{\mathcal{C}}^{-}$, l must be to the left of r. In the first case y is to the right of l, but below r and therefore cannot be in between l and r. In the second case, y is to the left of r but above l and again cannot be in between l and r. Hence, the compactness of $\mathcal{C}$ is unaffected if y is missing from a cluster sequence. □

Corollary 3 *Let* $\{\Gamma^+, \Gamma^-\}$ *describe a floorplan with a compact cluster* $\mathcal{C}$. *Then any permutation of* Γ^+ *or* Γ^- *that (i) preserves ordering* $\prec$ *among the cluster modules, and (ii) does not put the same cluster gap module in both* $\mathcal{S}_{\mathcal{C}}^+$ *and* $\mathcal{S}_{\mathcal{C}}^-$, *will preserve the compactness of* $\mathcal{C}$.

Proof. By Theorem 3, any permutation that achieves (i) keeps the cluster intact. By Theorem 4, any permutation that achieves (ii) does not affect compactness of a cluster. Hence, any permutation that achieves both (i) and (ii) keeps a compact cluster intact. □

Up to this point we have developed theorems assuming the cluster was compact. What has not been done is establish those conditions that produce a compact cluster in the first place. First, it is important to understand the interpretation of the sequence-pair notation.

- $x' \in \mathcal{M}^{bb}(x) \Rightarrow x'$ is "to the left of" x
- $x' \in \mathcal{M}^{ba}(x) \Rightarrow x'$ is "above" x
- $x' \in \mathcal{M}^{ab}(x) \Rightarrow x'$ is "below" x
- $x' \in \mathcal{M}^{aa}(x) \Rightarrow x'$ is "to the right of" x

The main idea is to look at the cluster gap modules in Γ^+ and determine what is their appropriate location in Γ^- to ensure the cluster gap module is outside of the compact cluster. For example, suppose we have a compact cluster $\mathcal{C}$ with

$$\Gamma^+ = (\ldots\ x\ y\ z\ \ldots)$$

where $x, z \in \mathcal{C}$ and $y \notin \mathcal{C}$. Assume $x \prec z$ in both sequence-pairs so x is to the left of z. There are only three possible (relative) locations of y in Γ^-:

1. $\Gamma^- = (\ldots\ x\ y\ z\ \ldots) \Rightarrow y \in \mathcal{M}^{aa}(x)$ and $y \in \mathcal{M}^{bb}(z)$ (precluded by Theorem 4)
2. $\Gamma^- = (\ldots\ x\ z\ y\ \ldots) \Rightarrow y \in \mathcal{M}^{aa}(x)$ and $y \in \mathcal{M}^{ba}(z)$ (to the right of x and above z)
3. $\Gamma^- = (\ldots\ y\ x\ z\ \ldots) \Rightarrow y \in \mathcal{M}^{ab}(x)$ and $y \in \mathcal{M}^{bb}(z)$ (below x and to the left of z)

Hence, there are only two legitimate locations for y relative to the cluster modules. This suggests a very important locations principle:

> *There are only specific locations of cluster gap modules in* $\{\Gamma^+, \Gamma^-\}$ *that are legal and ensure a compact cluster remains intact.*

This has strong implications for stochastic search algorithms. Essentially this principle says that random permutations of sequence-pairs are likely to destroy a compact cluster. But there is also a suggested solution: encode only "legal" locations of cluster gap modules into the search operators. For instance, in simulated annealing a new candidate solution would be created by randomly selecting a module and randomly inserting it somewhere else in Γ^-. But where the module could be inserted depends on a set of heuristics. From the example above, this module could not be inserted in the middle of $\mathcal{S}_{\mathcal{C}}^-$ if it were already in $\mathcal{S}_{\mathcal{C}}^+$. This construction of an "intelligent" search operator could lead to far more computationally efficient searches for optimal floorplans where cluster integrity must be maintained because illegal floorplans would never be produced for evaluation.

For convenience we designate four cluster modules with the compass point labels north, south, east, or west. These four modules represent the outer limits of a compact cluster (see Figure 3). Note that is possible for one module to be designated with more than one compass point. Consider a set of K rectangular modules in a floorplan desribed by $\{\Gamma^+, \Gamma^-\}$. Let

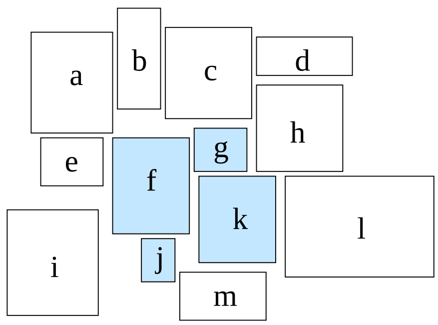

Figure 3: A floorplan with a compact cluster (the shaded modules). Module g is designated as the "north" module, k as the "east" module, m as the "south" module and f as the "west" module.

$\{a_1 a_2 \ldots a_r\} \in \mathcal{C}$ (a compact cluster) with $r << K$. There is a defined ordering $\prec$ in both $\mathcal{S}_{\mathcal{C}}^+$ and $\mathcal{S}_{\mathcal{C}}^-$. Let there be another module $y \notin \mathcal{C}$ with no restrictions on its location in Γ^+. We can define the following heuristic rules for the location of y in $\mathcal{S}_{\mathcal{C}}^-$:

Rule 1:

Rule 2:

Rule 3:

Rule 4:

Theorem 5 *The above heuristic rules are necessary (but not sufficient) to produce a compact cluster.*

Proof.