Representing Real Numbers with Binary Strings

Suppose you are using ℓ-bit binary strings to represent a real number $x \in [x_{\min}, x_{\max}]$. This can be done with precision $O(2^{-\ell})$.

Let $\triangle x = \frac{x_{\max} - x_{\min}}{2^{\ell}}$. Now consider an arbitrary binary string $\mathcal{B} \in \{0,1\}^{\ell}$. Its real number equivalent is

$$x \;=\; x_{\min} + \mathcal{B}_I \cdot \triangle\; x$$

where $\mathcal{B}_I$ is the integer equivalent of $\mathcal{B}$.

Example:

Let $x \in [-5, 5]$ be a number represented by a binary string of length $\ell = 8$. Consider the string

$$\mathcal{B} = \{b_7\, b_6\, \ldots b_0\} = 10010011.$$

This string represents the real number x, which can be computed as follows:

1. $\mathcal{B}_I = \sum_{k=0}^{7}\; b_i \cdot 2^k \;=\; 147$

2. $\triangle x = \frac{5-(-5)}{2^8} \approx 0.039$

3. $x = -5 + \mathcal{B}_I \cdot \triangle x \;=\; -5 + (147)(0.039) = 0.733$