Evolving Neural Networks for a Generalized Divide the Dollar Game

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Basic Divide the Dollar Game

Simplified version of John Nash's bargaining problem game.

Simple rules:

(1) n=2 players submit a bid on much of $1 they are willing to accept

(2) Compute the bid total = $\text{bid}_1 + \text{bid}_2$.

(3)
$$\text{payoff}(\mathit{player}_k) = \begin{cases} \text{bid}_k & \text{bid total} \leq \$1 \\ 0 & \text{bid total} > \$1 \end{cases}$$

Note: If only integer bids are allowed, then each pair of bids is a **Nash Equilibrium**.

Generalized Divide the Dollar Game

Originally proposed by the Ashlock & Greenwood[1].

Same game except

1. now is an n-player game ($n > 2$)
2. allows for small subsidies

With subsidies players can submit bids that total more than $1 and still get their bids because the excess is covered by the subsidy.

But these subsidies are bounded and available only when the bids are fair—i.e., roughly equal so no player is exploited.

[1] D. Ashlock and G. Greenwood, Generalized divide the dollar, in *IEEE 2016 Congress on Evolutionary Computation*, pages 343–350, 2016

Suppose a small subsidy of \$0.05 is available.

Example 1

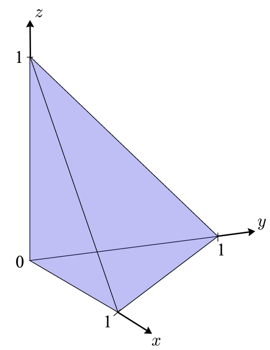
$n = 3$ players with bids $\{0.33, 0.36, 0.35\}$. Total over \$1, but bids are fair. Subsidy covers the excess.

Example 2

$n = 3$ players with bids $\{0.30, 0.72, 0.02\}$. Total over \$1, but no subsidy because of unfair bids.

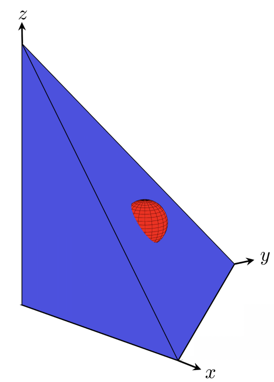
Generalized game can be envisioned as each player choosing a coordinate of a point in a subspace $S \subset \mathbb{R}^n$. Each coordinate represents a player's bid and the sum of the coordinates equals the sum of the bids.

Players are said to be **coordinated** if the bid total is $\leq$ \$1.



The triangle with vertices at (1 0 0), (0 1 0), and (0 0 1) is a 2-simplex. Every point on the simplex has a bid total of \$1.

All points on and beneath the simplex represent coordinated bids.



Subspace of interest for a 3-player generalized divide the dollar game with subsidies. The hemisphere protruding from the center of the 2-simplex shows the subsidy region.

Representation

definition

Representation $=$ genome $+$ move or search operator

Representation is important in all evolutionary algorithms because it determines how effective the algorithm will be in finding good solutions to a given problem.

The genome describes how solutions are encoded.

The move operator determines which solutions can be found during a search.

Example Representation

The genome space (G) contains all solutions and the search space (S) is what the EA explores. They are not necessarily the same.

Consider $i = 00\ldots0$ and $j = 00\ldots1$ in G.

Example 1

Move operator is "flip one bit". Then $i, j \in S$

Example 2

Move operator is "flip two distinct bits". If $i \in S$ then $j \notin S$.

Representation

In this work the genome was a neural network. The architecture was fixed and the move operator was used to find suitable synaptic weight values.

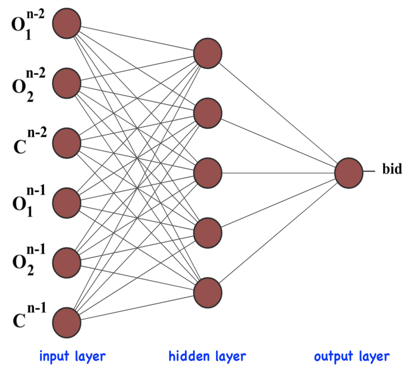
Two move operators were investigated:

1) differential evolution
2) evolution strategy[2]

Representation #1: NN + DE
Representation #2: NN + CMA-ES

[2] $(\mu/\mu, \lambda)$-ES with $\mu = 7, \lambda = 14$

NN architecture



sigmoid activation function

$$f(t) = \frac{2}{1 + e^{-5t}}$$

The output node maps the activation value to the unit interval using

$$O(t) = \frac{f(t) + 1}{2}$$

Inputs O_1 and O_2 represent bids made by the two opponents in the two previous time steps $n-1$ and $n-2$.

The input $C = \{0, 0.5\}$ denotes whether the respective pair of bids plus the bid by the evaluated NN resulted in a coordinated bid (0.5 if coordinated bid, 0 if not).

For example, the 3-tuple

$$\{O^{n-1}, O^{n-1}, C^{n-1}\} = \{0.22, 0.63, 0\}$$

says in the previous time step the two opponents bid 0.22 and 0.63 and the sum of the three bids was uncoordinated.

NN fitness

The population size was $NP = 36$ for both representations.

A NN was evaluated by competing in a series of 3-player, 15 round tournaments.

Each NN participated in a tournament against all possible pairs in the current population (no self play).

The payoffs were averaged over the 15 round tournament and then averaged again over the total number of tournaments (595) each NN competed in.

The payoff average over all tournaments is the fitness value of that NN.

Table: Tournament Outcomes (subsidy = 0)

Players	DE				CMA-ES			
	Bids			Total	Bids			Total
NN_1, NN_2, NN_3	0.32	0.29	0.31	0.92	0.33	0.32	0.32	0.97
NN_1, NN_2, NN_4	0.32	0.27	0.36	0.95	0.33	0.32	0.32	0.97
NN_1, NN_2, NN_5	0.21	0.17	0.33	0.71	0.33	0.32	0.32	0.97
NN_1, NN_3, NN_4	0.30	0.31	0.35	0.96	0.33	0.32	0.32	0.97
NN_1, NN_3, NN_5	0.31	0.31	0.31	0.93	0.33	0.32	0.32	0.97
NN_2, NN_3, NN_4	0.28	0.32	0.34	0.94	0.32	0.32	0.32	0.96
NN_2, NN_3, NN_5	0.29	0.32	0.31	0.92	0.32	0.32	0.32	0.96

NN_1 is the best fit NN, NN_2 the next best fit ....
All results shown after 100 generations

Table: Tournament Outcomes (subsidy = 0.10)

	DE				CMA-ES			
Players	Bids			Total	Bids			Total
NN_1, NN_2, NN_3	0.33	0.34	0.34	1.01	0.33	0.35	0.37	1.05
NN_1, NN_2, NN_4	0.34	0.33	0.31	0.98	0.34	0.35	0.34	1.03
NN_1, NN_2, NN_5	0.33	0.34	0.37	1.04	0.33	0.35	0.37	1.05
NN_1, NN_3, NN_4	0.34	0.33	0.31	0.98	0.33	0.37	0.35	1.05
NN_1, NN_3, NN_5	0.33	0.34	0.37	1.04	0.33	0.37	0.37	1.07
NN_2, NN_3, NN_4	0.33	0.34	0.31	0.98	0.35	0.37	0.34	1.06
NN_2, NN_3, NN_5	0.34	0.34	0.37	1.05	0.35	0.36	0.36	1.07

NN_1 is the best fit NN, NN_2 the next best fit ....
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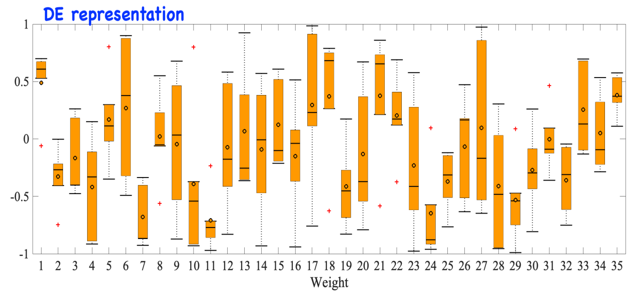
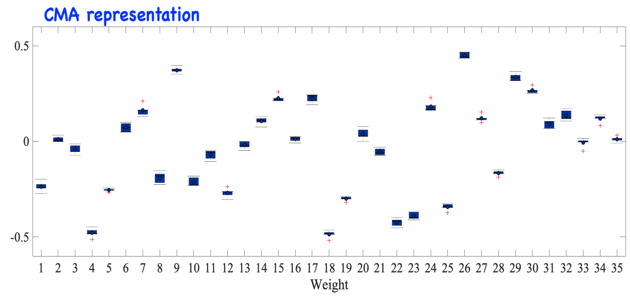
synaptic weight variation

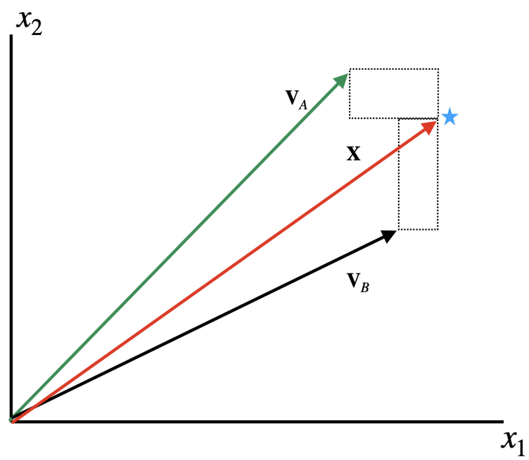




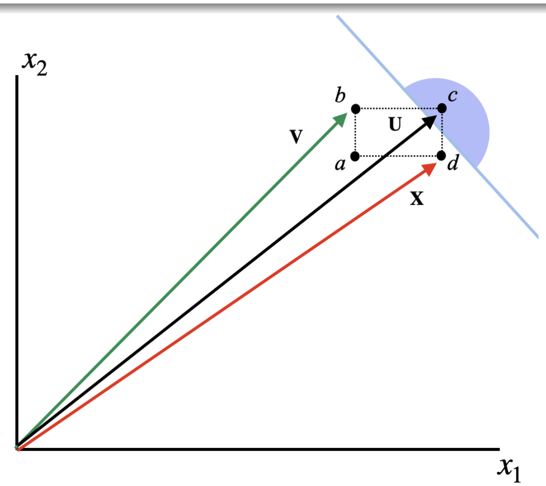
The CMA representation consistently did better than the DE representation. Why?

The answer lies in the way the move operator explores genotype space—i.e., how offspring are created.

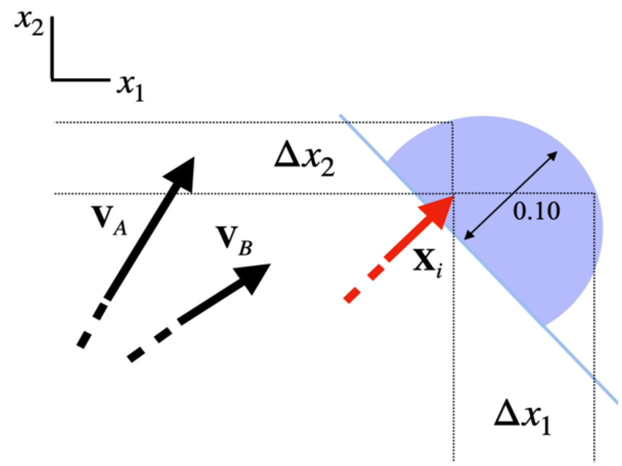
In the CMA-ES, the move operator is mutation, whereas in DE a combination of mutation and recombination.



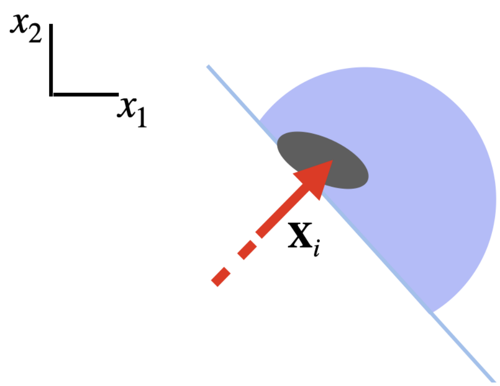
The crossover box location for two different donor vectors V_A and V_B. X is the target vector. A trial vector U will be located at one of the corners of the crossover box depending on which components of the donor and target vector are selected.



Creating a trial vector in the 2-d plane via crossover to exploit a subsidy. X is the target vector and V is a donor vector. Corners on the crossover box show possible locations for the trial vector U, *only one of which exploits the subsidy*. (The blue semi-circle is the subsidy region.)



Exploiting a subsidy requires proper placement of the crossover box, which requires donor vectors be in a very narrow range (Δx_1 and/or Δx_2). $\boldsymbol{V}_A$ creates a crossover box with a corner deeper inside the target region (blue area); $\boldsymbol{V}_B$ does not. $\boldsymbol{X}_i$ is the target vector.



CMA-ES reproduction. $\boldsymbol{X}_i$ is the parent and offspring are sampled mostly from the area within the dark gray ellipse.

Summary & Future Work

- We looked at two different representations (genome + move operator) for players in the generalized divide the dollar game.
- Genome was a multi-layer NN in both representations. Move operators were different: DE or CMA-ES.
- CMA-ES representation produced higher player bids, fair bids and better exploited subsidies
- Many real world business activities involve two or more people trying to reach a mutually beneficial agreement on some economic issue—i.e., negotiation among parties. Our tournament approach could be adapted to investigate negotiation strategies.
- Comparison against other representations (e.g., FSMs)

Gratzie!!