

Table of Continuous-time

Fourier Transform Pairs

$x(t) = \mathcal{F}_\omega^{-1}\{X(\omega)\} = \int_{-\infty}^{+\infty} x(t)e^{j\omega t}d\omega$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(\omega) = \mathcal{F}_\omega\{x(t)\} = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$
$x(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(\omega)$
$x(-t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(-\omega)$
$x^*(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X^*(-\omega)$
$x(t)$ is purely real	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(f) = X^*(-\omega)$ even/symmetry
$x(t)$ is purely imaginary	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(f) = -X^*(-\omega)$ odd/antisymmetry
even/symmetry $x(t) = x^*(-t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(\omega)$ is purely real
odd/antisymmetry $x(t) = -x^*(-t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(\omega)$ is purely imaginary
time shifting $x(t - t_0)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(\omega)e^{-j\omega t_0}$
$x(t)e^{j\omega_0 t}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(\omega - \omega_0)$ frequency shifting
time scaling $x(af)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{ a }X\left(\frac{\omega}{a}\right)$
$\frac{1}{ a }x\left(\frac{t}{a}\right)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X(a\omega)$ frequency scaling
$ax_1(t) + bx_2(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$aX_1(\omega) + bX_2(\omega)$
$x_1(t)x_2(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{2\pi}X_1(\omega) * X_2(\omega)$
$x_1(t) * x_2(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$X_1(\omega)X_2(\omega)$
$\delta(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	1
$\delta(t - t_0)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$e^{-j\omega t_0}$
1	$\xleftrightarrow{\mathcal{F}_\omega}$	$2\pi\delta(\omega)$
$e^{j\omega_0 t}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$2\pi\delta(\omega - \omega_0)$
$e^{-at}u(t) \quad a > 0$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{2a}{a^2 + \omega^2}$
$e^{-at}u(t) \quad \Re\{a\} > 0$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{a + j\omega}$
$e^{-at}u(-t) \quad \Re\{a\} > 0$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{a - j\omega}$
$e^{\frac{t^2}{2\sigma^2}}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\sigma\sqrt{2\pi}e^{-\frac{\sigma^2\omega^2}{2}}$
$\sin(\omega_0 t + \phi)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$j\pi [e^{-j\phi}\delta(\omega + \omega_0) - e^{j\phi}\delta(\omega - \omega_0)]$
$\cos(\omega_0 t + \phi)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\pi [e^{-j\phi}\delta(\omega + \omega_0) + e^{j\phi}\delta(\omega - \omega_0)]$
$x(t)\sin(\omega_0 t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{j}{2}[X(\omega + \omega_0) - X(\omega - \omega_0)]$
$x(t)\cos(\omega_0 t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{2}[X(\omega + \omega_0) + X(\omega - \omega_0)]$
$\sin^2(\omega_0 t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\pi^2 [2\delta(f) - \delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$
$\cos^2(\omega_0 t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\pi^2 [2\delta(\omega) + \delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$
$\text{rect}\left(\frac{t}{T}\right) = 1_{[-\frac{T}{2}, +\frac{T}{2}]}(t) = \begin{cases} 1 & t \leq \frac{T}{2} \\ 0 & t > \frac{T}{2} \end{cases}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$T \text{sinc}\left(\frac{\omega T}{2}\right)$
$\text{triang}\left(\frac{t}{T}\right) = \begin{cases} 1 - \frac{ t }{T} & t \leq T \\ 0 & t > T \end{cases}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$T \text{sinc}^2\left(\frac{\omega T}{2}\right)$
$u(t) = 1_{[0, +\infty)}(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\pi\delta(f) + \frac{1}{j\omega}$
$\text{sgn}(t) = \begin{cases} 1 & t \geq 0 \\ -1 & t < 0 \end{cases}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{2}{j\omega}$
$\text{sinc}(Tt)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{T} \text{rect}\left(\frac{\omega}{2\pi T}\right) = \frac{1}{T} 1_{[-2\pi T, +2\pi T]}(f)$
$\text{sinc}^2(Tt)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$\frac{1}{T} \text{triang}\left(\frac{\omega}{2\pi T}\right)$
$\frac{d^n}{dt^n} f(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$(j\omega)^n X(\omega)$
$t^n f(t)$	$\xleftrightarrow{\mathcal{F}_\omega}$	$j^n \frac{d^n}{df^n} X(\omega)$
$\frac{1}{t}$	$\xleftrightarrow{\mathcal{F}_\omega}$	$-j\pi \text{sgn}(\omega)$

Table of Discrete Time Fourier Transform (DTFT) Pairs

$x[n] = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X(e^{j\omega}) e^{j\omega n} d\omega$	$\xLeftrightarrow{DTFT}$	$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n}$
$x[n]$	$\xLeftrightarrow{DTFT}$	$X(e^{j\omega})$
$x[-n]$	$\xLeftrightarrow{DTFT}$	$X(e^{-j\omega})$
$x^*[n]$	$\xLeftrightarrow{DTFT}$	$X^*(e^{-j\omega})$
$x[n]$ is purely real	$\xLeftrightarrow{DTFT}$	$X(e^{j\omega}) = X^*(e^{-j\omega})$ even/symmetry
$x[n]$ is purely imaginary	$\xLeftrightarrow{DTFT}$	$X(e^{j\omega}) = -X^*(e^{-j\omega})$ odd/antisymmetry
even/symmetry $x[n] = x^*[-n]$	$\xLeftrightarrow{DTFT}$	$X(e^{j\omega})$ is purely real
odd/antisymmetry $x[n] = -x^*[-n]$	$\xLeftrightarrow{DTFT}$	$X(e^{j\omega})$ is purely imaginary
time shifting $x[n - n_0]$	$\xLeftrightarrow{DTFT}$	$X(e^{j\omega}) e^{-j\omega n_0}$
$x[n] e^{j\omega_0 n}$	$\xLeftrightarrow{DTFT}$	$X(e^{j(\omega - \omega_0)})$ frequency shifting
downsampling by N $x[Nn] \quad N \in \mathbb{N}_0$	$\xLeftrightarrow{DTFT}$	$\frac{1}{N} \sum_{k=0}^{N-1} X(e^{j\frac{\omega - 2\pi k}{N}})$
upsampling by N $\begin{cases} x[\frac{n}{N}] & n = kN \\ 0 & \text{otherwise} \end{cases}$	$\xLeftrightarrow{DTFT}$	$X(e^{jN\omega})$
$ax_1[n] + bx_2[n]$	$\xLeftrightarrow{DTFT}$	$aX_1(e^{j\omega}) + bX_2(e^{j\omega})$
$x_1[n]x_2[n]$	$\xLeftrightarrow{DTFT}$	$X_1(e^{j\omega}) * X_2(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} X_1(e^{j(\omega - \sigma)}) X_2(e^{j\sigma}) d\sigma$
$x_1[n] * x_2[n]$	$\xLeftrightarrow{DTFT}$	$X_1(e^{j\omega}) X_2(e^{j\omega})$
$\delta[n]$	$\xLeftrightarrow{DTFT}$	1
$\delta[n - n_0]$	$\xLeftrightarrow{DTFT}$	$e^{-j\omega n_0}$
1	$\xLeftrightarrow{DTFT}$	$\bar{\delta}(\omega) = \sum_{k=-\infty}^{+\infty} \delta(\omega + 2\pi k)$
$e^{j\omega_0 n}$	$\xLeftrightarrow{DTFT}$	$\bar{\delta}(\omega - \omega_0) = \sum_{k=-\infty}^{+\infty} \delta(\omega - \omega_0 + 2\pi k)$
$u[n]$	$\xLeftrightarrow{DTFT}$	$\frac{1}{1 - e^{-j\omega}} + \frac{1}{2} \bar{\delta}(\omega)$
$a^n u[n] \quad (a < 1)$	$\xLeftrightarrow{DTFT}$	$\frac{1}{1 - ae^{-j\omega}}$
$(n+1)a^n u[n]$	$\xLeftrightarrow{DTFT}$	$\frac{1}{(1 - ae^{-j\omega})^2}$
$\sin(\omega_0 n + \phi)$	$\xLeftrightarrow{DTFT}$	$\frac{j}{2} [e^{-j\phi} \bar{\delta}(\omega + \omega_0 + 2\pi k) - e^{+j\phi} \bar{\delta}(\omega - \omega_0 + 2\pi k)]$
$\cos(\omega_0 n + \phi)$	$\xLeftrightarrow{DTFT}$	$\frac{1}{2} [e^{-j\phi} \bar{\delta}(\omega + \omega_0 + 2\pi k) + e^{+j\phi} \bar{\delta}(\omega - \omega_0 + 2\pi k)]$
$\frac{\sin(\omega_c n)}{n} = \omega_c \text{sinc}(\omega_c n)$	$\xLeftrightarrow{DTFT}$	$\text{rect}\left(\frac{\omega}{\omega_c}\right) = \begin{cases} 1 & \omega < \omega_c \\ 0 & \omega_c < \omega < \pi \end{cases}$
Window : $\text{rect}\left(\frac{n}{M}\right) = \begin{cases} 1 & n \leq M \\ 0 & \text{otherwise} \end{cases}$	$\xLeftrightarrow{DTFT}$	$\frac{\sin[\omega(M + \frac{1}{2})]}{\sin(\omega/2)}$
MA : $\text{rect}\left(\frac{n}{M} - \frac{1}{2}\right) = \begin{cases} 1 & 0 \leq n \leq M \\ 0 & \text{otherwise} \end{cases}$	$\xLeftrightarrow{DTFT}$	$\frac{\sin[\omega(M+1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
MA : $\text{rect}\left(\frac{n}{M-1} - \frac{1}{2}\right) = \begin{cases} 1 & 0 \leq n \leq M-1 \\ 0 & \text{otherwise} \end{cases}$	$\xLeftrightarrow{DTFT}$	$\frac{\sin[\omega M/2]}{\sin(\omega/2)} e^{-j\omega(M-1)/2}$
$nx[n]$	$\xLeftrightarrow{DTFT}$	$j \frac{d}{d\omega} X(e^{j\omega})$
$x[n] - x[n-1]$	$\xLeftrightarrow{DTFT}$	$(1 - e^{-j\omega}) X(e^{j\omega})$
$\frac{a^n \sin[\omega_0(n+1)]}{\sin \omega_0} u[n] \quad a < 1$	$\xLeftrightarrow{DTFT}$	$\frac{1}{1 - 2a \cos(\omega_0 e^{-j\omega}) + a^2 e^{-j2\omega}}$

Some remarks

$$\bar{\delta}(\omega) = \sum_{k=-\infty}^{+\infty} \delta(\omega + 2\pi k)$$

Parseval :

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{-\pi}^{+\pi} |X(e^{j\omega})|^2 d\omega$$

$$\text{rect}(\omega) = \sum_{k=-\infty}^{+\infty} \text{rect}(\omega + 2\pi k)$$

Table of Z-Transform Pairs

$x[n] = \mathcal{Z}^{-1}\{X(z)\} = \frac{1}{2\pi j} \oint X(z)z^{n-1}dz$		$\xleftrightarrow{\mathcal{Z}}$	$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{+\infty} x[n]z^{-n}$	ROC
$x[n]$		$\xleftrightarrow{\mathcal{Z}}$	$X(z)$	R_x
$x[-n]$		$\xleftrightarrow{\mathcal{Z}}$	$X(\frac{1}{z})$	$\frac{1}{R_x}$
$x^*[n]$		$\xleftrightarrow{\mathcal{Z}}$	$X^*(z^*)$	R_x
$x^*[-n]$		$\xleftrightarrow{\mathcal{Z}}$	$X^*(\frac{1}{z^*})$	$\frac{1}{R_x}$
$\Re\{x[n]\}$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	R_x
$\Im\{x[n]\}$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	R_x
time shifting $x[n - n_0]$		$\xleftrightarrow{\mathcal{Z}}$	$z^{-n_0}X(z)$	R_x
$a^n x[n]$		$\xleftrightarrow{\mathcal{Z}}$	$X(\frac{z}{a})$	$ a R_x$
downsampling by N $x[Nn]$ $N \in \mathbb{N}_0$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{1}{N} \sum_{k=0}^{N-1} X(W_N^k z^{\frac{1}{N}})$ $W_N = e^{-j\frac{2\pi}{N}}$	R_x
$ax_1[n] + bx_2[n]$		$\xleftrightarrow{\mathcal{Z}}$	$aX_1(z) + bX_2(z)$	$R_x \cap R_y$
$x_1[n]x_2[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{1}{2\pi j} \oint X_1(u)X_2(\frac{z}{u})u^{-1}du$	$R_x \cap R_y$
$x_1[n] * x_2[n]$		$\xleftrightarrow{\mathcal{Z}}$	$X_1(z)X_2(z)$	$R_x \cap R_y$
$\delta[n]$		$\xleftrightarrow{\mathcal{Z}}$	1	$\forall z$
$\delta[n - n_0]$		$\xleftrightarrow{\mathcal{Z}}$	z^{-n_0}	$\forall z$
$u[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{z-1}$	$ z > 1$
$-u[-n-1]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{z-1}$	$ z < 1$
$nu[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{(z-1)^2}$	$ z > 1$
$n^2u[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{z(z+1)}{(z-1)^3}$	$ z > 1$
$n^3u[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{z(z^2+4z+1)}{(z-1)^4}$	$ z > 1$
$(-1)^n$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{z+1}$	$ z < 1$
$a^n u[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{z-a}$	$ z > a $
$-a^n u[-n-1]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{z-a}$	$ z < a $
$a^{n-1}u[n-1]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{1}{z-a}$	$ z > a $
$na^n u[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{az}{(z-a)^2}$	$ z > a $
$n^2a^n u[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{az(z+a)}{(z-a)^3}$	$ z > a $
$e^{-an}u[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{z-e^{-a}}$	$ z > e^{-a} $
$\begin{cases} a^n & n = 0, \dots, N-1 \\ 0 & \text{otherwise} \end{cases}$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{1-a^N z^{-N}}{1-az^{-1}}$	$ z > 0$
$\sin(\omega_0 n)u[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{z \sin(\omega_0)}{z^2 - 2 \cos(\omega_0)z + 1}$	$ z > 1$
$\cos(\omega_0 n)u[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{z(z - \cos(\omega_0))}{z^2 - 2 \cos(\omega_0)z + 1}$	$ z > 1$
$a^n \sin(\omega_0 n)u[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{za \sin(\omega_0)}{z^2 - 2a \cos(\omega_0)z + a^2}$	$ z > a$
$a^n \cos(\omega_0 n)u[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{z(z - a \cos(\omega_0))}{z^2 - 2a \cos(\omega_0)z + a^2}$	$ z > a$
$nx[n]$		$\xleftrightarrow{\mathcal{Z}}$	$-z \frac{d}{dz} X(z)$	R_x
$\frac{x[n]}{n}$		$\xleftrightarrow{\mathcal{Z}}$	$-\int_0^z \frac{X(z)}{z} dz$	R_x
$\frac{\prod_{i=1}^m (n-i+1)}{a^m m!} a^m u[n]$		$\xleftrightarrow{\mathcal{Z}}$	$\frac{z}{(z-a)^{m+1}}$	

Please note : $\frac{z}{z-1} = \frac{z^{-1}}{1-z^{-1}}$