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TITLE OF PROPOSED PROJECT **Unitary Operators and Quantum Computing: A Study of Operator Eigenstructure**

REQUESTED AMOUNT	PROPOSED DURATION (1-60 MONTHS)	REQUESTED STARTING DATE	SHOW RELATED PRELIMINARY PROPOSAL NO. IF APPLICABLE
$ **49,574**	**12** months	**01/01/05**	

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CERTIFICATION PAGE

Certification for Authorized Organizational Representative or Individual Applicant:

By signing and submitting this proposal, the individual applicant or the authorized official of the applicant institution is: (1) certifying that statements made herein are true and complete to the best of his/her knowledge; and (2) agreeing to accept the obligation to comply with NSF award terms and conditions if an award is made as a result of this application. Further, the applicant is hereby providing certifications regarding debarment and suspension, drug-free workplace, and lobbying activities (see below), as set forth in Grant Proposal Guide (GPG), NSF 04-23. Willful provision of false information in this application and its supporting documents or in reports required under an ensuing award is a criminal offense (U. S. Code, Title 18, Section 1001).

In addition, if the applicant institution employs more than fifty persons, the authorized official of the applicant institution is certifying that the institution has implemented a written and enforced conflict of interest policy that is consistent with the provisions of Grant Policy Manual Section 510; that to the best of his/her knowledge, all financial disclosures required by that conflict of interest policy have been made; and that all identified conflicts of interest will have been satisfactorily managed, reduced or eliminated prior to the institution's expenditure of any funds under the award, in accordance with the institution's conflict of interest policy. Conflicts which cannot be satisfactorily managed, reduced or eliminated must be disclosed to NSF.

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A Framework for Conducting Eigenanalysis on Unitary Operators Used in Quantum Optimization Algorithms

(Summary)

Consider an NP-hard optimization problem f. This can be solved on a quantum computer by forming a superposition of all possible states and then applying a unitary operator U_f that amplifies the amplitude of the optimal solution while attenuating all other amplitudes. An observation operator can then extract the final answer.

The key to obtaining the optimal solution with a high probability is to have a properly constructed U_f. This unitary operator should permit interference to alter the amplitudes, but it is not known for certain what if any role entanglement should play. The eigenvalues of U_f all have modulus one, but little is known about what type of eigenstructure works best for a particular f. Moreover, it is unlikely that a different type of optimization problem g will have $U_g = U_f$. A deeper understanding of the eigenstructure of unitary operators is necessary before procedures can be developed to construct a correctly performing U_f for an arbitrary optimization problem. Eigenanalysis methods do exist, but there are few examples of properly operating unitary operators—i.e., operators designed to solve particular NP-hard problems—available for study. The purpose of this research project is correct this situation by creating a framework to support eigenanalysis investigations on unitary operators.

We believe the only practical way of obtaining these U_f's is to work with optimization problems small enough so that exhaustive search can find the optimal solution. This means the specific amplitude that must be amplified by U_f will be known *a priori* so it will be easy to check if a candidate U_f produces the desired effects. An evolutionary algorithm—i.e., a stochastic algorithm that conducts searches using the Darwinian principles of natural selection found in Nature—will be used to find a correctly operating U_f.

The eigenanalysis begins after the evolutionary algorithm has found an acceptable unitary operator U_f. Conventional methods (e.g., LAPACK routines) can be used to extract the eigenvalues and eigenvectors. We know the eigenvalues are of the form $\lambda_k = e^{i\theta_k}$, but we do not know how the θ_k's should be distributed for a specific U_f. The extent of degeneracy is another unknown. One of our particular interests is to investigate the role entanglement plays. (The eigenvalue spectra alone cannot prove state separability, but eigenvector positioning may prove to be useful.) The investigation will then explore how problem size affects the operator eigenstructure. We expect the eigenanalysis will enable us to identify intrinsic spectral properties of efficacious unitary operators. Our objectives can be summarized as follows:

Research Objectives: *(1) construct unitary operators that optimally solve instances of NP-hard problems, (2) apply LAPACK routines to conduct an eigenanalysis on these operators to determine their degree of eigenvalue degeneracy, their eigenvalue distribution and any role of entanglement, and (3) identify heuristics for constructing efficacious unitary operators for other types of quantum optimizationalgorithms.*

This 11 month project involves one PI and one graduate assistant. The project cost is approximately $46,000.

Statement on Intellectual Merit: The proposed research addresses an open problem that will have immense interest in the quantum computing community. Currently there are only a handful of known quantum algorithms and the results of this research will aid in the development of future algorithms.

The PI is a recognized expert in evolutionary computation, which is a key component of this research effort. The PI has also published a peer-reviewed paper on quantum computing and has given two seminars at Portland State University on the benefits of quantum computing.

Statement on Broader Impact: The results of this research will be widely disseminated with a web page where researchers can download all source code. Results will be initially released in the Los Alamos Preprint Archive and later published in a relevant journal.

The Portland State University Office of Educational Equity Programs and Services will help us to identify a qualified underrepresented group graduate student to work on this research project.

TABLE OF CONTENTS

For font size and page formatting specifications, see GPG section II.C.

Appendix Items:

*Proposers may select any numbering mechanism for the proposal. The entire proposal however, must be paginated. Complete both columns only if the proposal is numbered consecutively.

A Framework for Conducting Eigenanalysis on Unitary Operators for Quantum Optimization Algorithms

C.1 Introduction

The real challenge in solving optimization problems is to create algorithms and techniques that can solve realistically sized problems within a reasonable amount of computational time. Most of these algorithms formulate an optimization problem as a search problem—i.e., the problem solutions reside in an abstract solution space and two solutions are neighbors if they differ by a small perturbation of a problem parameter. Any algorithm that "solves" an optimization problem is therefore a search algorithm that explores the solution space landscape.

Unfortunately, many real-world optimization problems require such huge computational resources that brute force search methods are useless; they simply take too much time to find the optimal answer. This has led researchers to use search heuristics that yield an acceptable compromise: a possibly lower quality answer but with a minimal search effort. Recently an entirely new approach has surfaced with potentially enormous consequences. This new approach is called *quantum computing* and it relies on the principles of quantum mechanics to find problem solutions.

We are interested in solving optimization problems which have their solutions encoded as binary strings. This covers a broad class of problems including many which are NP-hard[1]. In principle, a classical computer takes an initial solution binary string and, using logic operations, transforms it into the final solution binary string. (Which specific logical operations are dictated by the search algorithm steps.) Since any logical operation can be implemented with logic gates, one could physically implement the search algorithm as a logic circuit composed of interconnected elementary logic gates.

This classical system perspective has been adopted by many developers of quantum computing search algorithms. Quantum mechanical systems evolve according to Schrödinger's equation—i.e., the initial system state is transformed into a final state by a series of unitary operations. Since problem solutions are encoded in quantum computers as a set of qubits, these unitary operators are usually defined as elementary quantum "gates" (e.g., a controlled-NOT gate). Although different optimization problems may use qubits to represent solutions, each optimization problem instance requires an entirely new "quantum circuit". This is because the qubit states that represent the optimal solution to one type of optimization problem will most likely *not* be the same for the optimal solution to a different optimization problem. Furthermore, it is not hard to see that the complexity of such a circuit, in terms of the number of interconnected gates, will be extremely high for even moderately sized problems. Consequently, a gate-level approach will rapidly become unmanageable and a higher level of abstraction is needed.

We believe it is better to use a system-level approach where the focus is not at the qubit level—the level

[1] The IEEE 754 standard describes how to encode floating point numbers as 32-bit binary strings. Hence, optimization problems with continuous search spaces are also of interest.

quantum gates work at—but rather at the state level. This requires direct manipulation of state amplitudes. Each distinct state represents a unique problem solution and our goal is therefore to amplify the amplitude of the one state that encodes the globally optimum solution to an optimization problem while attenuating the amplitudes of all the other states.

Unitary operators can alter the amplitudes, but it is not known how to construct such an operator to work for a specific optimization problem. In other words, a unitary operator that performs correctly for an instance of an **INDEPENDENT SET PROBLEM** will most likely not work for an instance of a **MULTIPROCESSOR SCHEDULING PROBLEM**—even though both problems encode their solutions with qubits[2]. What is clearly needed is a deeper understanding of the characteristics of a unitary operator that works for a specific optimization problem. This insight may ultimately lead to a practical method for developing effective unitary operators for other types of quantum algorithms. Our research objectives are listed below.

Research Objectives: *(1) construct unitary operators that optimally solve instances of NP-hard problems, (2) apply LAPACK routines to conduct an eigenanalysis on these operators to determine their degree of eigenvalue degeneracy, their eigenvalue distribution and any role of entanglement, and (3) identify heuristics for constructing efficacious unitary operators for other types of quantum algorithms.*

We intend to use *evolutionary algorithms*—i.e., algorithms that conduct searches using the principles of Darwinian evolution found in Nature—to design unitary operators. We will pick an optimization test problem small enough so that its optimal solution can be found via exhaustive search, making it possible to specify what the relative amplitudes of all states should be. Hence, it will be easy to determine if the unitary operator is correctly constructed. An eigenanalysis of the unitary operator can then be conducted. This study will be repeated for other NP-hard optimization problems to gain some insight into the characteristics of efficacious unitary operators.

This proposal is organized as follows. Section C.2 provides some needed background. The quantum computing overview is especially brief because its primary purpose here is to establish notation. The description of evolutionary algorithms should be sufficient to illustrate our methods. Section C.3 provides details on our approach. Finally, Section C.4 discusses broader impacts of this research effort.

C.2 Background

C.2.1 Quantum Computing

Classical computer systems represent a single bit of information deterministically: the value is either a logic 0 or a logic 1. Quantum computer systems represent a single bit of information as a *qubit*, which is a unit vector in a complex Hilbert space C^2. The ideas are commonly expressed using the bra/ket notation

[2]Both problems are known to be NP-hard [1].

introduced by Dirac [2]. The *ket* symbol is denoted by $|x\rangle$ and the corresponding *bra* is denoted by $\langle x|$. The ket describes a quantum state and the corresponding bra is its complex conjugate.

Any practical quantum computer manipulates a register of n qubits. If each qubit has an orthonormal basis $\{|0\rangle, |1\rangle\}$, then a n qubit system has a basis expressed by the *tensor product*: $C^2 \otimes C^2 \otimes \cdots C^2$. This gives 2^n total basis vectors. In general, $|a\rangle$ denotes the tensor product $|a_n\rangle \otimes |a_{n-1}\rangle \otimes \cdots \otimes |a_1\rangle \otimes |a_0\rangle$, which means a quantum register has the value $a = 2^0 a_0 + 2^1 a_1 + \cdots + 2^n a_n$.

A qubit need not exist in only one basis state. Indeed, a qubit can exist as a *linear superposition* of basis states $c_0|0\rangle + c_1|1\rangle$, where c_0, c_1 are complex numbers with $|c_0|^2 + |c_1|^2 = 1$. More generally, the n qubit register can be prepared in a superposition of all possible classical states:

$$|x\rangle \;=\; \sum_{i=0}^{2^n-1} c_i|i\rangle \tag{1}$$

where the normalization condition $\sum_i |c_i|^2 = 1$ must hold. The complex number c_i is called the *amplitude* associated with the state $|i\rangle$.

The state of a qubit register is determined by a measurement. In quantum systems this measurement process projects the system state onto one of the basis states. Referring to Eq. (1), the measurement returns a value of $|i\rangle$ with probability $|c_i|^2$. Any subsequent measurement returns the state $|i\rangle$ with probability 1, which means the measurement process irreversibly alters the state of the system. Measurement also gives another perspective on entanglement: two qubits are entangled if and only if the measurement of one effects the state of the other.

The most conventional representation of a base state $|i\rangle$ is as a column matrix with the $i-th$ entry 1 and all other entries 0. A state $|\psi\rangle$ is therefore represented as a column matrix of the complex amplitudes. That is,

$$|\psi\rangle \;=\; \begin{pmatrix} c_0 \\ c_1 \\ c_2 \\ \vdots \end{pmatrix}$$

Quantum systems evolve from state to state according to Schrödinger's equation [3]. Suppose we start in state $|\psi\rangle = \sum c_i|i\rangle$. A linear operator U produces a new state $|\phi\rangle = U|\psi\rangle$. Both states are linear combinations of the same base states, so $|\phi\rangle = \sum c_i{}'|i\rangle$. This means evolution occurs by modification of the state amplitudes. Note that the normalization condition required of states is satisfied iff U is unitary—i.e., $U^\dagger U = I$.

It is important to emphasize the role superposition plays in quantum computing. Consider a state $|\psi\rangle = \sum c_i|i\rangle$. You can exploit the superposition using the property of *quantum interference*. Interference allows the exponential number of computations performed in parallel to either cancel or enhance each other. Feynman [3] beautifully describes how light waves can constructively or destructively interfere to produce this effect. The goal of any quantum algorithm is to have a similar phenomena occur—i.e., interference increases the amplitudes of computational results we desire and decreases the amplitudes of the remaining results. It is a unitary operator that would alter these amplitudes.

C.2.2 Evolutionary Algorithms

All evolutionary algorithms (EAs) share the same basic organization: iterations of competitive selection and random variation. Although there are several varieties of EAs, they are all biologically inspired and generally follow the format depicted in Figure 1.

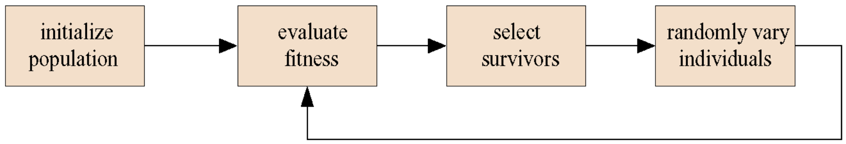

Figure 1: The canonical EA.

EAs manipulate a population of individuals where each distinct individual encodes a unique set of problem parameters needed to form a solution. The initial population is randomly generated. During each generation (iteration), the current population is evaluated and each individual is assigned a numerical fitness value. High fitness means the associated individual represents a good solution to the given problem. The selection process chooses the higher fit individuals for reproduction. These survivors undergo stochastic reproduction operations to create new individuals. The loop shown in Figure 1 continues until either a fixed number of generations are processed or an acceptable solution has been found.

The three basic EA paradigms used for optimization are the *genetic algorithm* (GA), the *evolution strategy* (ES), and *evolutionary programming* (EP). Each paradigm was independently developed. Although they all follow the evolving population model, there are some differences. For example, the GA chooses parents with a probability proportional to its fitness with respect to other individuals in the current population. This means the fitter parents are more frequently chosen for reproduction. Conversely, ES and EP allow every parent to reproduce regardless of its relative fitness. ES ranks all parents and offspring according to fitness and deterministically chooses the best to be parents in the next generation; EP conducts a tournament among all individuals and the tournament outcome determines who survives. Most importantly, GAs use components from two parents to produce offspring—a process called recombination—as the primary reproduction operator, whereas EP only uses mutation. The ES can use multi-parent recombination, but it relies heavily on mutation for reproduction.

The No Free Lunch Theorem [4] states that no search algorithm works best over all classes of optimization problems. We therefore plan on constructing a hybrid algorithm that adopts features from all three EA paradigms, which makes the term “evolutionary algorithm” an appropriate nomenclature.

C.3 Outline of Research Activity

C.3.1 The Goal of the Research Project

We intend to use a system-level approach that dispenses with a quantum gate-level viewpoint of quantum algorithms. First, recall an optimization problem f is solved on a quantum computer by the following quantum algorithm:

1. encode each possible solution with n qubits
2. form a superposition of all possible solutions
3. apply a unitary operator U_f that amplifies the amplitude of the optimal solution while attenuating all other amplitudes
4. take an observation to extract the final answer.

Each distinct optimization problem should intuitively have a unique unitary operator—i.e., if f and g are two optimization problems, in general $U_f \neq U_g$ unless $f = g$. The problem is how to construct a unitary operator tailored for a specific optimization problem. Unfortunately, no one really knows how to do this construction which explains why little progress has been made in designing optimization algorithms for quantum computers.

The goal of this research project is to construct a correctly operating U_f for a (small) NP-hard problem. This operator runs on a classical computer. Nevertheless, the research community benefits in two ways from having an actual, properly functioning U_f:

1. Insight often comes from observing what works. Quantum algorithm designers will now have a functioning U_f to study. They will now know what type of operators their design method must create.
2. The U_f is a benchmark that can be used to compare different quantum algorithm design methods.

We believe the only practical way of obtaining these U_f's is to work with optimization problems small enough so that exhaustive search can find the optimal solution. This means the specific amplitude that must be amplified by U_f will be known *a priori* making it possible to easily check if a candidate U_f produces the desired effects. Exhaustive search on small size NP-hard problems has been used by others to verify the efficacy of quantum algorithms [5].

We will create a framework that creates a properly operating unitary operators for study. Our framework contains an NP-hard optimization problem f, and a stochastic search method for creating a suitable U_f. Conventional methods that rely on proven, publicly available software routines can then be used to actually conduct the eigenanalysis.

C.3.1.1 Details on the NP-hard Problem

C.3.1.1.1 Problem Definition

One good choice for an optimization problem is the **INDEPENDENT SET PROBLEM (IS)** which is known to be NP-hard [1]. The problem is defined as follows:

Problem instance: A graph $G = (V, E)$ where $V = \{1, 2, \ldots, n\}$ is the set of vertices and $E \subseteq V \times V$ the set of edges. An edge between vertices i , j is denoted by the pair $(i\ , j) \in E$.
Feasible solution: A set V' of nodes such that $\forall\ i\ , j \in V' : (i\ , j) \notin E$. V' is called an *independent set.*
Optimal solution: maximal $|V'|$—i.e., the max cardinality independent set

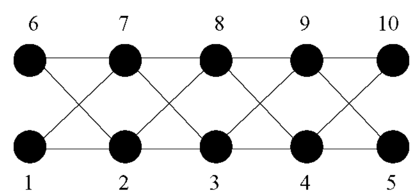

Figure 2: An **IS** problem instance. Both $V_1' = \{2, 4, 7, 9\}$ and $V_2' = \{1, 6, 9\}$ are independent sets, but neither one is globally optimum.

C.3.1.1.2 Exhaustive Search for the Optimal Solution

Consider an **IS** instance with ℓ nodes. Each solution could be encoded for a classical computer using an ℓ-bit binary string where bit $i = 1$ indicates $i \in V'$. (Not all of these solutions are feasible though.)

We need to find the optimal solution so we will know which amplitude to amplify. It is easy to verify if a solution is feasible, so the main problem is to list all of the solutions. Fortunately this is trival to do for **IS** instances because the solutions are encoded as binary strings: simply count in binary from 0 to 2^ℓ-1. It does take $O(2^n)$ time to scan for the maximal cardinality, but the problem size is purposely kept small enough so that this can be done within a reasonable timeframe.

C.3.2 Details on the Evolutionary Algorithm

An ℓ-node **IS** problem solution can be encoded with ℓ qubits. Thus, a quantum computer state $|\psi\rangle$ would be a linear superposition of all 2^ℓ base states, and $|\psi\rangle$ would be represented as a column matrix of complex amplitudes.

Each individual in the EA's population encodes a candidate unitary matrix. The initial population of unitary matrices must be randomly created but this can be efficiently done [6]. The basic idea is to make the unitary $2^\ell \times 2^\ell$ matrix U a block diagonal matrix. Each 2×2 block submatrix represents an elementary unitary transformation, which are functions of the random angles α, ϕ, and χ each with a uniform distribution on $[0,2\pi)$. A random initial population is then formed by choosing random angle values for each member of the population. A column matrix representing an initial state must also be created; each entry c_i is complex and of equal modulus subject to the constraint $\sum |c_i|^2 = 1$.

New candidate unitary matrices must be created for evaluation during each generation of the EA. The technique described in [6] can be also be used as a reproduction operator: take an existing unitary matrix and perturb each random angle as follows

$$\alpha' = \alpha + N(0, \sigma_1) \qquad \phi' = \phi + N(0, \sigma_2) \qquad \chi' = \chi + N(0, \sigma_3)$$

where $N(0, \sigma)$ represents a Normally distributed random variable with zero mean and standard deviation σ. Note that each random angle has its own standard deviation. These standard deviations can also be adapted to improve the search. This method of stochastic mutation to produce new candidate solutions is commonly used with EAs and has been shown to be a very effective search mechanism [7].

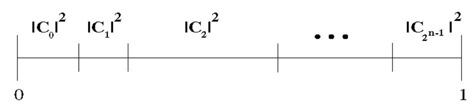

Figure 3: Each amplitude is allocated a "bin" on the unit interval with the bin's width equal to the amplitude squared. An observation is made by picking a random number on the unit interval and recording the bin number. States with the larger amplitudes have a greater probability of being picked.

The optimization problem is small enough so that an exhaustive search can find the optimal solution $|i_{\text{opt}}\rangle$. Consequently, the specific amplitude we want to maximize will be known *a priori*. An EA will attempt to evolve a unitary operator U_f that amplifies c_{opt} while attenuating all other amplitudes. Specifically, during each generation the EA takes every candidate U_f from the current population and computes $|\phi\rangle = U_f|\psi\rangle$. The fitness of U_f indicates how well the amplitude of the optimal solution has been amplified with respect to the other solutions—i.e., highly fit solutions have $|c_{\text{opt}}|^2 > |c_i|^2 \; \forall \; |i\rangle \neq |i_{\text{opt}}\rangle$.

The fitness calculation is straightforward. The user defines a lower bound for $|c_{\text{opt}}|^2$ and an upper bound for all other amplitudes. After computing $|\phi\rangle = U_f|\psi\rangle$, the complex amplitudes of $c_i \in |\phi\rangle$ are extracted and the fitness is calculated as follows

$$\text{fitness}(U_f) \;=\; \sum_i \frac{1}{[|c_i|^2 - |c_i^*|^2]^2 + \epsilon}$$

where $\epsilon \ll 1$ and $|c_i^*|^2$ is the defined upper or lower amplitude bounds as appropriate. The EA terminates whenever the maximum fitness $1/\epsilon$ is reached or a fixed number of generations have been processed. An observation can then be taken (see Figure 3). In practice, a number of such observations will have to be taken to construct a probability density.

C.3.3 Conducting the Eigenanalysis

The eigenanalysis begins after the EA has found an acceptable unitary operator U_f. Conventional methods (e.g., LAPACK routines) can be used to extract the eigenvalues and eigenvectors. We know the eigenvalues are of the form $\lambda_k = e^{i\theta_k}$, but we do not know how the θ_k's should be distributed for a specific U_f. The extent of degeneracy is another unknown. One of our particular interests is to investigate the role entanglement plays. (The eigenvalue spectra alone cannot prove state separability [8], but eigenvector positioning may prove to be useful [9].) We will see how problem size affects the unitary operator eigenstructure.

A characterization of the "fitness landscape" near the global optimum must be completed before attaching any significance to the eigenanalysis results. More formally, a fitness landscape consists of

- a large (albeit finite) set of solutions S
- a fitness function $F : S \rightarrow \Re_+$ (the positive real number line)
- the concept of a neighborhood between solutions

A landscape is considered "smooth" in the region surrounding a particular solution if its neighboring points—i.e., solutions with nearly identical parameter values—differ in fitness by only a small amount. Conversely, a landscape is "rugged" if its neighboring points differ markedly in fitness. A promising statistical approach to characterizing landscapes was advanced by Weinberger [10] who suggested using a random walk to gather statistical information. Starting at some randomly chosen solution s_a the walk next visits a randomly chosen neighbor. Repeating this process yields a sequence of fitness values $F_a, F_{a+1}, \ldots$. Weinberger assumed that since there is some underlying distribution of fitness values, a random walk in any direction is sufficient to gather statistics. The degree of correlation between two solutions t steps apart in this random walk is given by the correlation function

$$R(t) = \frac{\langle F_a F_{a+t} \rangle - \langle F_a \rangle^2}{\sigma_f^2}$$

where $\langle \cdot \rangle$ means the expected value over all pairs t steps apart. If a high degree of correlation exists, then the landscape is smooth. Highly uncorrelated landscapes have a large number of local optima and any adaptive walk (i.e., a walk restricted to fitter neighbors) is likely to stop very quickly Kauffman [11]. These landscapes are presumed to be *statistically isotropic*. In other words, independent of where the random walk begins, the statistical information is invariant; a sufficiently long walk will infer any correlation present in the landscape. Since we are only interested in the landscape structure near the global optimum, a random walk restricted to an open ball centered at the global optimum will be sufficient to measure the correlation [12].

The NP problems of interest have markedly different fitness landscapes. Hence, it should be possible to establish a correspondence between fitness landscape structure (at least in the neighborhood of the global optimum) and eigenvalue distribution and placement. The results of our eigenanalysis and landscape analysis will help identify methods for constructing unitary operators for other types of quantum algorithms. We will publish these methods as a guide for other quantum algorithm designers.

C.4 Broader Impact of Research Effort

C.4.1 Broad Dissemination to Enhance Scientific Understanding

A summary of this research will be distributed in several diverse venues. In addition to the prompt publication of these results in an appropriate journal (e.g., Physical Review), we intend to initially release the results in the Los Alamos Physics Preprint Archive. We also intend to establish a web site providing open access to all results. All publications related to this research, including preprints, will contain the website URL. The URL will also be posted on evolutionary computation bulletin boards such as GAList. All source code will be downloadable from the web site.

C.4.2 Representation of Underrepresented Groups

The budget only asks for funding of a graduate student, but some of the work (primarily the LAPACK runs needed to collect data for later analysis) can be done with undergraduate students. We will attempt to provide financial support independent of this NSF program by pursuing supplemental federal grants that target undergraduate research.

Portland State University is located in downtown Portland, Oregon. This university prides itself on its diverse student population. We therefore anticipate having a large pool of traditionally underrepresented students to choose from to work on this project. Portland State University has an Office of Educational Equity Programs and Services (EPPS) who helps to provide educational assistance (scholarships, academic advising, mentoring, etc.) to traditionally underrepresented students. We have contacted this office and they have agreed to help us identify qualified graduate and undergraduate students to work on this research project. In the mean time, I am a graduate advisor to two female students in our department. I will actively try to recruit one of them to work on this project.

References

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[11] S. Kauffman. *The Origins of Order: Self-Organization and Selection in Evolution.* Oxford Univ. Press, 1993.

[12] G. Greenwood and X. Hu. Are landscapes for constrained optimization problems statistically isotropic? *Physica Scripta*, 57:321–323, 1998.

BIOGRAPHICAL SKETCH

Garrison W. Greenwood

a. Professional Preparation

Undergraduate Institution	Calif. Polytechnic St. Univ.	BS (Elec. Engr.) 1976
Graduate Institution	Calif. Polytechnic St. Univ.	MS (Elec. Engr.) 1978
	Univ. of Washington	Ph.D. (Elec. Engr.) 1992

b. Appointments

Portland State University	Portland, OR
Western Michigan University	Kalamazoo, MI
SpaceLabs Medical, Inc.	Redmond, WA
Voice Computer Corp.	Redmond, WA
Sundstrand Data Control, Inc.	Redmond, WA
Eldec Corp.	Lynnwood, WA
Honeywell, Inc.	Mukilteo, WA
Boeing, Inc.	Seattle, WA
Naval Weapons Station	Seal Beach, CA

c. Publications

(*i*) Related Publications

1. G. Greenwood, "Finding Solutions to NP Problems: Philosophical Differences Between Quantum and Evolutionary Search Algorithms", *Proc. Congress Evol. Comput.*, 815-822, 2001
2. G. Greenwood and X. Hu, "On the use of random walks to estimate correlation in fitness landscapes", *Computational Statistics & Data Analysis*, Vol 28, No. 2, 131-137, 1998
3. G. Greenwood and X. Hu, "Are landscapes for constrained optimization problems statistically isotropic?", *Physica Scripta*, Vol. 57, 321-323, 1998

(*ii*) Other Publications

1. G. Greenwood and Q. Zhu, "Convergence in Evolutionary Programs with Self-Adaptation", *Evolutionary Computation* Vol. 9, No. 2, 147-158, 2001
2. G. Greenwood, "Revisiting the complexity of finding globally minimum energy configurations in atomic clusters", *Zeitschrift für Physikalische Chemie*, Vol. 211, 105-114, 1999
3. G. Greenwood, "Chaotic behavior in evolution strategies", *Physica D* 109 (3-4), 343-350, 1997

d. Synergistic Activities

1. Twelve years of full-time work experience in private industry as an electronic design engineer. Primary responsibility was the hardware design of multiprocessor, embedded computer systems.
2. Currently serving as associate editor of the IEEE Transactions on Evolutionary Computation.
3. Nine years of full-time teaching experience in electronic engineering.
4. Served on IGERT proposal review panel at NSF (1999).

e. Collaborators & Other Affiliations

(*i*) Collaborators (last 48 months)

Burcin Aktan (Intel Corp.), Danny Chen (University of Notre Dame), Malgorzata Chrzanowska-Jeske (Portland State University), Manuel Ciobanu (Western Michigan University), Gary Fogel (Natural Selection Inc., San Diego, CA), Ajay Gupta (Western Michigan University), Xiaobo (Sharon) Hu (University of Notre Dame), Damon Miller (Western Michigan University), Sai Ravichandran (Western Michigan University), Molly Shor (Oregon State University), Xiaoyu Song (Portland State University), Gang Quan (University of Notre Dame), Benyi Wang (Portland State University)

(*ii*) Ph.D. Graduate Advisor

Arun Somani (currently at Iowa State University)

(*iii*) Thesis Advisor

served as MS thesis advisor for the following students while at Western Michigan University: Sai Ravichandran, Min Chen, Christian Lang

SUMMARY PROPOSAL BUDGET YEAR 1

ORGANIZATION Portland State University	FOR NSF USE ONLY	PROPOSAL NO.	DURATION (months) Proposed	Granted	
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR Garrison Greenwood		AWARD NO.			

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. Garrison Greenwood - none	0.00	0.00	1.50	$ 10,830	$
2.					
3.					
4.					
5.					
6. (0) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	1.50	10,830	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL ASSOCIATES	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (1) GRADUATE STUDENTS				10,980	
4. (0) UNDERGRADUATE STUDENTS				0	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				21,810	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				3,040	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				24,850	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.) computer purchase $ 2,000					
TOTAL EQUIPMENT				2,000	
E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS)				2,000	
2. FOREIGN				0	
F. PARTICIPANT SUPPORT COSTS 1. STIPENDS $ 0 2. TRAVEL 0 3. SUBSISTENCE 0 4. OTHER 6,588					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				6,588	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				0	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				0	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				0	
TOTAL OTHER DIRECT COSTS				0	
H. TOTAL DIRECT COSTS (A THROUGH G)				35,438	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE) TMDC (Rate: 49.0000, Base: 28849)					
TOTAL INDIRECT COSTS (F&A)				14,136	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				49,574	
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPG II.C.6.j.)				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 49,574	$
M. COST SHARING PROPOSED LEVEL $ 0	AGREED LEVEL IF DIFFERENT $				

PI/PD NAME Garrison Greenwood	FOR NSF USE ONLY INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME*	Date Checked	Date Of Rate Sheet	Initials - ORG

 *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

SUMMARY PROPOSAL BUDGET

Cumulative

ORGANIZATION: Portland State University				FOR NSF USE ONLY — PROPOSAL NO.	DURATION (months): Proposed / Granted
PRINCIPAL INVESTIGATOR / PROJECT DIRECTOR: Garrison Greenwood				AWARD NO.	

A. SENIOR PERSONNEL: PI/PD, Co-PI's, Faculty and Other Senior Associates (List each separately with title, A.7. show number in brackets)	NSF Funded Person-months CAL	ACAD	SUMR	Funds Requested By proposer	Funds granted by NSF (if different)
1. Garrison Greenwood - none	0.00	0.00	1.50	$ 10,830	$
2.					
3.					
4.					
5.					
6. ( ) OTHERS (LIST INDIVIDUALLY ON BUDGET JUSTIFICATION PAGE)	0.00	0.00	0.00	0	
7. (1) TOTAL SENIOR PERSONNEL (1 - 6)	0.00	0.00	1.50	10,830	
B. OTHER PERSONNEL (SHOW NUMBERS IN BRACKETS)					
1. (0) POST DOCTORAL ASSOCIATES	0.00	0.00	0.00	0	
2. (0) OTHER PROFESSIONALS (TECHNICIAN, PROGRAMMER, ETC.)	0.00	0.00	0.00	0	
3. (1) GRADUATE STUDENTS				10,980	
4. (0) UNDERGRADUATE STUDENTS				0	
5. (0) SECRETARIAL - CLERICAL (IF CHARGED DIRECTLY)				0	
6. (0) OTHER				0	
TOTAL SALARIES AND WAGES (A + B)				21,810	
C. FRINGE BENEFITS (IF CHARGED AS DIRECT COSTS)				3,040	
TOTAL SALARIES, WAGES AND FRINGE BENEFITS (A + B + C)				24,850	
D. EQUIPMENT (LIST ITEM AND DOLLAR AMOUNT FOR EACH ITEM EXCEEDING $5,000.) $ 2,000					
TOTAL EQUIPMENT				2,000	
E. TRAVEL 1. DOMESTIC (INCL. CANADA, MEXICO AND U.S. POSSESSIONS)				2,000	
2. FOREIGN				0	
F. PARTICIPANT SUPPORT COSTS					
1. STIPENDS $ 0					
2. TRAVEL 0					
3. SUBSISTENCE 0					
4. OTHER 6,588					
TOTAL NUMBER OF PARTICIPANTS (0) TOTAL PARTICIPANT COSTS				6,588	
G. OTHER DIRECT COSTS					
1. MATERIALS AND SUPPLIES				0	
2. PUBLICATION COSTS/DOCUMENTATION/DISSEMINATION				0	
3. CONSULTANT SERVICES				0	
4. COMPUTER SERVICES				0	
5. SUBAWARDS				0	
6. OTHER				0	
TOTAL OTHER DIRECT COSTS				0	
H. TOTAL DIRECT COSTS (A THROUGH G)				35,438	
I. INDIRECT COSTS (F&A)(SPECIFY RATE AND BASE)					
TOTAL INDIRECT COSTS (F&A)				14,136	
J. TOTAL DIRECT AND INDIRECT COSTS (H + I)				49,574	
K. RESIDUAL FUNDS (IF FOR FURTHER SUPPORT OF CURRENT PROJECTS SEE GPG II.C.6.j.)				0	
L. AMOUNT OF THIS REQUEST (J) OR (J MINUS K)				$ 49,574	$
M. COST SHARING PROPOSED LEVEL $ 0	AGREED LEVEL IF DIFFERENT $				

PI/PD NAME: Garrison Greenwood	FOR NSF USE ONLY — INDIRECT COST RATE VERIFICATION		
ORG. REP. NAME*	Date Checked	Date Of Rate Sheet	Initials - ORG

C *ELECTRONIC SIGNATURES REQUIRED FOR REVISED BUDGET

Budget Justification Page

Fringe benefit costs based on 23% of PI summer salary and 5% academic year salary for graduate student.

Travel is to attend one technical conference to present results.

Computer purchase is for PC to run simulations.

Tuition remission is not included in indirect costs.