ECE 315 HW #2. (Due 24 Jan)

Each problem worth 3 points.

1) Show a block diagram of a system characterized by the ODE

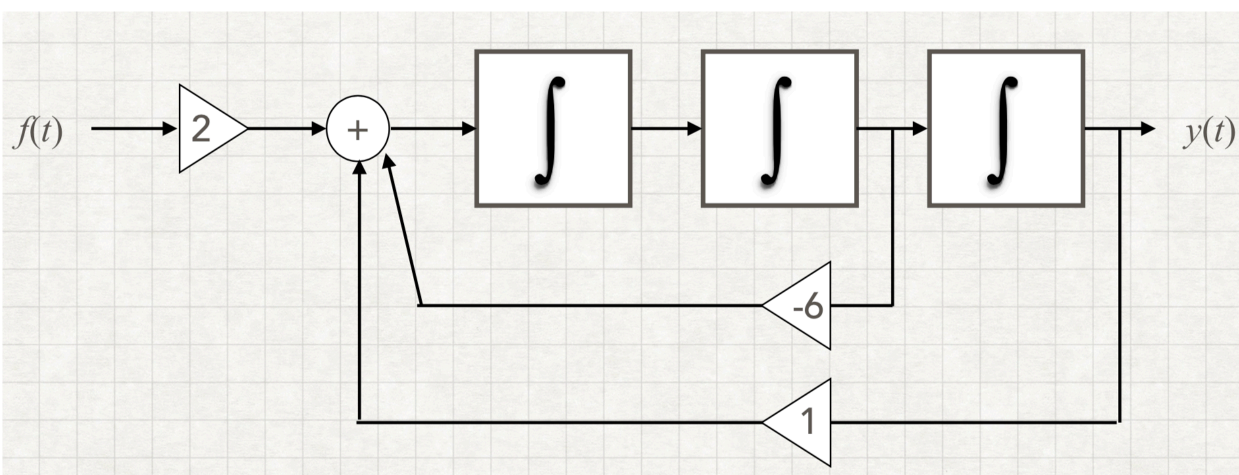
$$\frac{d^3y(t)}{dt^3} + 6\frac{dy(t)}{dt} - y(t) = 2\frac{dx(t)}{dt}$$

2) A system input-output relations is $y(t) = ax(t) + b$ where $x(t)$ is the input, $y(t)$ the output, and a and b are constants. Is this system linear? (Justify)

3) Prove the system shown in Figure 4.11 in the textbook is LTI.

SOLUTIONS

1)



2) If $b \neq 0$, then the system is not linear because $x = 0$ implies $y = b \neq 0$. If $b = 0$, then the system is linear.

3) additivity: let $y_1 = \int x_1(t)\,dt$ and $y_2 = \int x_2(t)\,dt$. Then

$$\int \left(x_1(t) + x_2(t)\right) dt = \int x_1(t)\,dt + \int x_2(t)\,dt = y_1(t) + y_2(t)$$

Because the integral of a sum is the sum of the separate integrals.

homogenity:

$$\int a x_1(t)\,dt = a \int x_1(t)\,dt = a y_1(t)$$

Since both additivity and homogenity hold, the system is linear.

NOTE: Both of these integral properties can be deduced from the Riemann sum approximation to integration. In the Riemann sum formula,

$$\int_a^b f(x)\,dx = \lim_{\Delta x_k \to 0} \sum_{k=1}^{N} f(x_k^*) \Delta x_k$$

Where $a \leq x \leq b$ and x_k^* is an arbitrary point in the interval Δx_k

And no, I don't expect you make any reference to the Riemann sum to get full credit for this problem. If you just show the integral properties for additivity and scaling that is good enough