ECE 312 HW #6 Soln

(1)

$$Z\{x[n-k]\} = \sum_{n=-\infty}^{\infty} x[n-k] z^{-n}$$

let $m = n-k \Rightarrow n = m+k$

$n = -\infty \Rightarrow m = -\infty$
$n = \infty \Rightarrow m = \infty$

substituting,

$$\sum_{m=-\infty}^{\infty} x[m] z^{-m-k}$$

$$= \sum_{m=-\infty}^{\infty} x[m] z^{-m} z^{-k}$$

$$= z^{-k} \sum_{m=-\infty}^{\infty} x[m] z^{-m}$$

$$= z^{-k} X(z)$$

(2) (a) $\mathcal{Z}\{u[n]\} = \dfrac{1}{1-z^{-1}}$

$\therefore\ \mathcal{Z}\{u[n-k]\} = \dfrac{z^{-k}}{1-z^{-1}}$

ROC: $|z| > 1$

(b) $\mathcal{Z}\{(0.2)^n\} = \dfrac{1}{1-(0.2)z^{-1}}$ for $(0.2)^n$ causal

ROC: $|z| > |a|$

(c) $\mathcal{Z}\{\cos \Omega_0 n\} = \dfrac{z^2-(\cos\Omega_0)z}{z^2-(2\cos\Omega_0)z+1}$

$\cos\Omega_0 n$ causal and $\Omega_0 = \pi/4$

$\therefore\ \mathcal{Z}\{\cos n\pi/4\} = \dfrac{z^2-(\cos\pi/4)z}{z^2-(2\cos\pi/4)z+1}$

ROC: $|z| > 1$

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$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n}$$

$$= \sum_{n=-2}^{3} x[n] z^{-n}$$

$$= -2z^{2} + z + 4 + 2z^{-2} - 5z^{-3}$$

ROC: for $z \neq 0$ or $z \neq \infty$
each term is finite
$\Downarrow$
series converges

$\therefore \quad 0 < |z| < \infty$

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