ECE 312 HW #4 solutions

1)

A system has an impulse response $\mathrm{h}(t) = 4e^{-4t}\,\mathrm{u}(t)$. Find and plot the response of the system to the excitation $\mathrm{x}(t) = \mathrm{rect}(2(t-1/4))$.

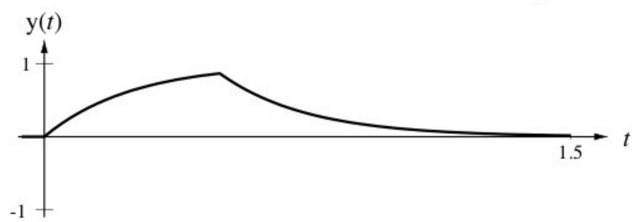
$$\mathrm{y}(t) = \mathrm{x}(t) * \mathrm{h}(t) = \mathrm{rect}(2(t-1/4)) * 4e^{-4t}\,\mathrm{u}(t) = 4(\mathrm{u}(t) - \mathrm{u}(t-1/2)) * e^{-4t}\,\mathrm{u}(t)$$

$$\mathrm{u}(t) * e^{-4t}\,\mathrm{u}(t) = \int_{-\infty}^{\infty} \mathrm{u}(\tau) e^{-4(t-\tau)}\,\mathrm{u}(t-\tau)\,d\tau = \int_0^t e^{-4(t-\tau)} d\tau(t)$$

$$\mathrm{u}(t) * e^{-4t}\,\mathrm{u}(t) = \begin{Bmatrix} (1/4)(1-e^{-4t}) & , t > 0 \\ 0 & , t < 0 \end{Bmatrix} = \frac{1}{4}(1-e^{-4t})\mathrm{u}(t)$$

Invoking linearity and time-invariance,

$$\mathrm{y}(t) = 4(\mathrm{u}(t) - \mathrm{u}(t-1/2)) * e^{-4t}\,\mathrm{u}(t) = (1-e^{-4t})\mathrm{u}(t) - \left(1-e^{-4(t-1/2)}\right)\mathrm{u}(t-1/2)$$



2)

(a) $\mathrm{x}(t) = 4\,\mathrm{rect}(4t) * \delta_1(t)$

$$\mathrm{c_x}[k] = \frac{1}{T_0}\int_{T_0} \mathrm{x}(t) e^{-j2\pi(kf_0)t}\,dt = 4\int_{-\frac{1}{2}}^{\frac{1}{2}} \mathrm{rect}(4t) e^{-j2\pi kt}\,dt = 4\int_{-\frac{1}{8}}^{\frac{1}{8}} e^{-j2\pi kt}\,dt$$

$$\mathrm{c_x}[k] = 4\left[\frac{e^{-j2\pi kt}}{-j2\pi k}\right]_{-\frac{1}{8}}^{\frac{1}{8}} = \frac{4}{\pi k}\left(\frac{e^{-j\frac{\pi k}{4}} - e^{+j\frac{\pi k}{4}}}{-j2}\right) = \frac{4}{\pi k}\sin\left(\frac{\pi k}{4}\right) = \mathrm{sinc}\left(\frac{k}{4}\right)$$

3)

The CTFS harmonic function of $\mathrm{x}(t)$ based on one fundamental period is found to be

$$\mathrm{c_x}[k] = \frac{1-\cos(\pi k)}{(\pi k)^2}$$

(a) Is the signal even, odd or neither?

Since the harmonic function is purely real, the signal is even.

(b) What is the numerical average value of the signal?

The average value of the signal is $\mathrm{c_x}[0]$. Since the form is indeterminate for $n=0$, one must apply L Hôpital's rule.

$$\mathrm{c_x}[0] = \lim_{k\to 0} \frac{1-\cos(\pi k)}{(\pi k)^2} = \lim_{k\to 0} \frac{\pi \sin(\pi k)}{2n\pi^2} = \lim_{k\to 0} \frac{\pi^2 \cos(\pi k)}{2\pi^2} = \frac{1}{2}$$

4)

$$c_k = \frac{Ae^{\frac{-jk\pi}{2}}}{2} \frac{\sin(\frac{k\pi}{2})}{(\frac{k\pi}{2})}$$