ECE 312 HW #3

① (a)

$$y_h = c_1 e^{-t} + c_2 e^{-4t}$$

$$y_h' = -c_1 e^{-t} - 4c_2 e^{-4t}$$

$$y_h'' = c_1 e^{-t} + 16 c_2 e^{-4t}$$

substituting,

$$c_1 e^{-t} + 16 c_2 e^{-4t} - 5c_1 e^{-t} - 20 c_2 e^{-4t}$$
$$+ 4c_1 e^{-t} + 4c_2 e^{-4t} = 0$$

$$e^{-t}[c_1 - 5c_1 + 4c_1]$$
$$+ e^{-4t}[16c_2 - 20c_2 + 4c_2] = 0$$

$$y_p = K \quad (\text{a const.})$$

$$y_p' = y_p'' = 0$$

$$4K = 1 \Rightarrow K = 1/4$$

$$y(t) = y_h(t) + y_p(t)$$

$$= c_1 e^{-t} + c_2 e^{-4t} + 1/4$$

$$y(0) = c_1 + c_2 + 1/4 = 0$$

$$y'(t) = -c_1 - 4c_2 = 0$$

$$c_1 + c_2 = -1/4$$
$$-c_1 - 4c_2 = 0$$
$$-3c_2 = -1/4 \Rightarrow c_2 = 1/12$$
$$c_1 = -1/3$$

$$\therefore \quad y(t) = -\frac{1}{3}e^{-t} + \frac{1}{12}e^{-4t} + \frac{1}{4}$$

(b) impulse response is

$$\hat{y}(t) = \frac{dy(t)}{dt} \quad \text{(from part (a))}$$

$$\therefore \quad \hat{y}(t) = \frac{1}{3}e^{-t} - \frac{1}{3}e^{-4t}$$

(c)
$$s^2Y(s) + 5sY(s) + 4Y(s) = X(s)$$
$$Y(s)\left[s^2 + 5s + 4\right] = X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2 + 5s + 4}$$

(2)

$$x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

In the 2nd integral let $\alpha = t - \tau$

then $\tau = t - \alpha$ when $\tau = -\infty, \alpha = \infty$

$d\alpha = -d\tau$ $\tau = +\infty, \alpha = -\infty$

Substituting

$$h(t) * x(t) = -\int_{\infty}^{-\infty} h(t-\alpha) x(\alpha) d\alpha$$

$$= \int_{-\infty}^{\infty} h(t-\alpha) x(\alpha) d\alpha$$

Q.E.D.

3. $h(t) = \left(e^{-3t}\cos 5t\right)u(t)$

$$y(t) = \int_{-\infty}^{t} h(\tau)\,d\tau$$

$$= \int_{0}^{t} e^{-3\tau}\cos 5\tau\,d\tau$$

from the integral tables on the XML file,

$$y(t) = \left.\frac{e^{-3\tau}(5\sin 5\tau - 3\cos 5\tau)}{9+25}\right|_{0}^{t}$$

$$= \frac{e^{-3t}(5\sin 5t - 3\cos 5t)}{34} + \frac{3}{34}$$

4. Plot should be an isocelesis triangle