ECE 312 HW #2

1. (4 points) problem 6, page 153. (do the nonlinearity and time invariance checks only)

Homogeneity:

Let $x_1(t) = g(t)$. Then $y_1(t) = u(g(t))$.

Let $x_2(t) = K g(t)$. Then $y_2(t) = u(K g(t)) \neq K y_1(t) = K u(g(t))$.

Not homogeneous

Additivity:

Let $x_1(t) = g(t)$. Then $y_1(t) = u(g(t))$.

Let $x_2(t) = h(t)$. Then $y_2(t) = u(h(t))$.

Let $x_3(t) = g(t) + h(t)$.

Then $y_3(t) = u(g(t) + h(t)) \neq y_1(t) + y_2(t) = u(g(t)) + u(h(t))$

Not additive

Since it is not homogeneous and not additive, it is not linear.

Let $x_1(t) = g(t)$. Then $y_1(t) = u(g(t))$.

Let $x_2(t) = g(t - t_0)$.

Then $y_2(t) = u(g(t - t_0)) = y_1(t - t_0)$.

Time Invariant

2. (3 points) problem 7, page 153 (do the linearity check only)

Homogeneity:
Let $x_1(t) = g(t)$. Then $y_1(t) = g(t-5) - g(3-t)$.
Let $x_2(t) = K\,g(t)$. Then $y_2(t) = K\,g(t-5) - K\,g(3-t) = K\,y_1(t)$.
Homogeneous

Additivity:
Let $x_1(t) = g(t)$. Then $y_1(t) = g(t-5) - g(3-t)$.
Let $x_2(t) = h(t)$. Then $y_2(t) = h(t-5) - h(3-t)$.
Let $x_3(t) = g(t) + h(t)$.
Then $y_3(t) = g(t-5) + h(t-5) - g(3-t) - h(3-t) = y_1(t) + y_2(t)$
Additive

Since it is both homogeneous and additive, it is also linear.

3. (3 points) a system is described by

$$y(t) = \int_{-\infty}^{t} x(\mu) d\mu$$

is this system linear? is it time-invariant?

Let

$$y_1(t) = \int_{-\infty}^{t} x_1(\mu)\, d\mu \quad \text{and} \quad y_2(t) = \int_{-\infty}^{t} x_2(\mu)\, d\mu$$

Note the following:

$$\int_{-\infty}^{t} Ax_2(\mu)\, d\mu \;=\; A\int_{-\infty}^{t} x_2(\mu)\, d\mu \;=\; A\, y_2(t)$$

and

$$\int_{-\infty}^{t} [x_1(\mu) + x_2(\mu)]\, d\mu \;=\; \int_{-\infty}^{t} x_1(\mu)\, d\mu + \int_{-\infty}^{t} x_2(\mu)\, d\mu \;=\; y_1(t) + y_2(t)$$

Therefore it is linear.

Consider

$$\int_{-\infty}^{t} x(\mu - t_0)\, d\mu$$

where t_0 is a constant. Let $b = \mu - t_0$. Then db = dμ and the limits are

$$\mu = -\infty \;\Rightarrow\; b = -\infty$$
$$\mu = t \;\Rightarrow\; b = t - t_0$$

The integral this becomes

$$\int_{-\infty}^{t} x(\mu - t_0)\, d\mu \;=\; \int_{-\infty}^{t-t_0} x(b)\, db = y(t - t_0)$$

So it is time invariant.