ECE 312 Exam #2 Soln

Each question is worth 10 points.

1. Determine if each of the following signals is period:

(a) $x[n] = \cos\frac{1}{4}n$ no $\rightarrow \cos\Omega_0 n$ with $\Omega_0 = \frac{1}{4} \Rightarrow \frac{\Omega_0}{2\pi} = \frac{1}{8\pi}$ *

(b) $x[n] = e^{j(\pi/4)n}$ yes $\rightarrow e^{j\frac{\pi}{4}n} = e^{j\Omega_0 n}$ with $\frac{\Omega_0}{2\pi} = \frac{1}{8}$ ‡

(c) $x[n] = \sum_{k=-\infty}^{\infty} \delta[n-5k]$ yes

(d) $x[n] = e^{-2n}u[n]$ no

(e) $x[n] = 1 + 0.7\sin(2\pi n)$ no $\rightarrow x[n] = 1 \ \forall n$

* not a rational number

‡ is a rational number

2. Consider a causal LTI discrete system governed by the difference equation

$$y[n]+0.8y[n-1]+y[n-2]=x[n]$$

(a) Find the frequency response $H(e^{j\omega})$ (just give the equation)

(b) Find at least one eigenvalue. (you need only provide the equation for them since you are not allowed to have a calculator during the exam.)

(c) What is $h[n]$ for $n=0,\ 1,\ 2,\ 3$?

(d) What is the unit step response for n = 0, 1, 2, 3?

(a) $$Y(z) + 0.8z^{-1}Y(z) + z^{-2}Y(z) = X(z)$$

$$\left[1+0.8z^{-1}+z^{-2}\right] Y(z) = X(z)$$

$$\therefore\ H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1+0.8z^{-1}+z^{-2}}$$

$$H(e^{jw}) = \frac{1}{1+0.8e^{-jw}+e^{-j2w}}$$

(b) $$z = \frac{-0.8 \pm \sqrt{(0.8)^2-4}}{2}$$

(c)

n	$x[n]$	$h[n]$
0	1	1
1	0	-0.8
2	0	$(-0.8)^2-1=-0.36$
3	0	$(-0.8)(-0.36)+0.8 = 1.088$

$$y[n] = -0.8y[n-1] - y[n-2] + x[n]$$

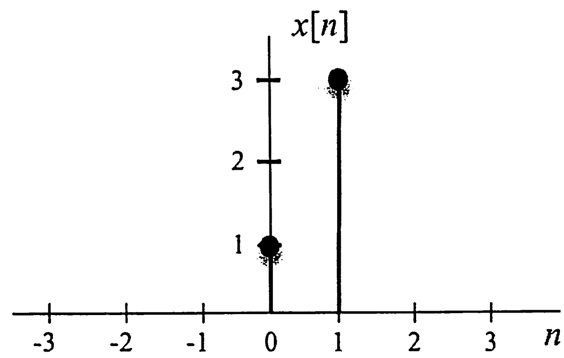
(d) $y[0] = h[0] = 1$

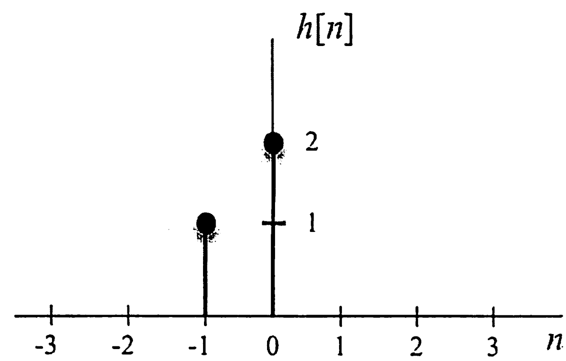
$y[1] = h[0]+h[1] = 0.2$

$y[2] = h[0]+h[1]+h[2] = -0.16$

$y[3] = h[0]+h[1]+h[2]+h[3] = 0.928$

3. A LTI system has the following input ($x[n]$) and impulse response ($h[n]$):





(a) (3 points) Use convolution to find the system output response $y[n]$ for $n = 0$, 1, 2, 3, 4 and 5.

(b) (2 points) Is $h[n]$ causal? (explain)

(c) (5 points) In class it was stated that all physically realizable systems (i.e., systems you can actually build) must be causal. Why? (hint: use your results from part (a).)

(a) $y[0] = h[0] \cdot x[0] + h[-1] \cdot x[1] = 5$

$y[1] = h[0] \cdot x[1] = 6$

$y[2] = y[3] = y[4] = y[5] = 0$

(b) no because $h[-1] \neq 0$

(c) if $h[n]$ is not causal, you must know future $x[n]$ values to compute the current $y[n]$ value

4. Either a DFT or a CTFS could be used to get the spectrum of a periodic signal. Yet in practice the DFT is far, far more often to be used. Why?

To get the CTFS coefficients (the harmonics) you must know the equation of the periodic signal. Usually you don't know it.

DFT only requires samples of the signal, which can always be done

5. A 4-point DFT produced the following harmonics:

$$X[0]=1,\ \ X[1]=2,\ \ X[2]=\ 1+j,\ \ X[3]=1-j$$

What is the energy in $x[n]$?

$$E = \sum_{n=0}^{3} |x[n]|^2$$

$$= 1^2 + 2^2 + (1+j)(1-j)$$

$$+ (1-j)(1+j)$$

$$= 1 + 4 + 2 + 2 = 9$$