Solution

ECE 312 Exam #1

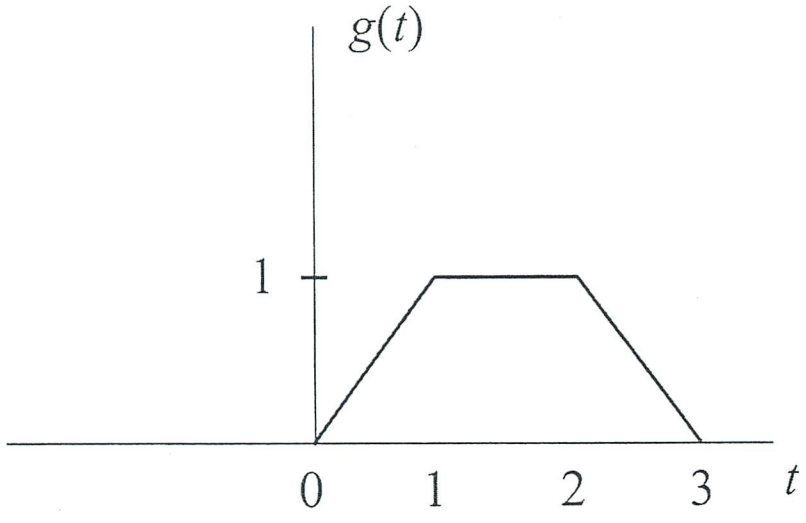
ALL ANSWERS MUST BE ON THE EXAM SHEETS.

1. (10 points) The spectrum of a periodic signal $x(t)$ can be obtained with either a CTFS or a DFT. However, in practice engineers almost always use the DFT. Why?

CTFS requires solving an integral to get the harmonics (Fourier coefficients). You need to know the equation of $x(t)$ to solve the integral. Often not the case

DFT requires samples of $x(t)$ to get the harmonics. You can always sample a signal

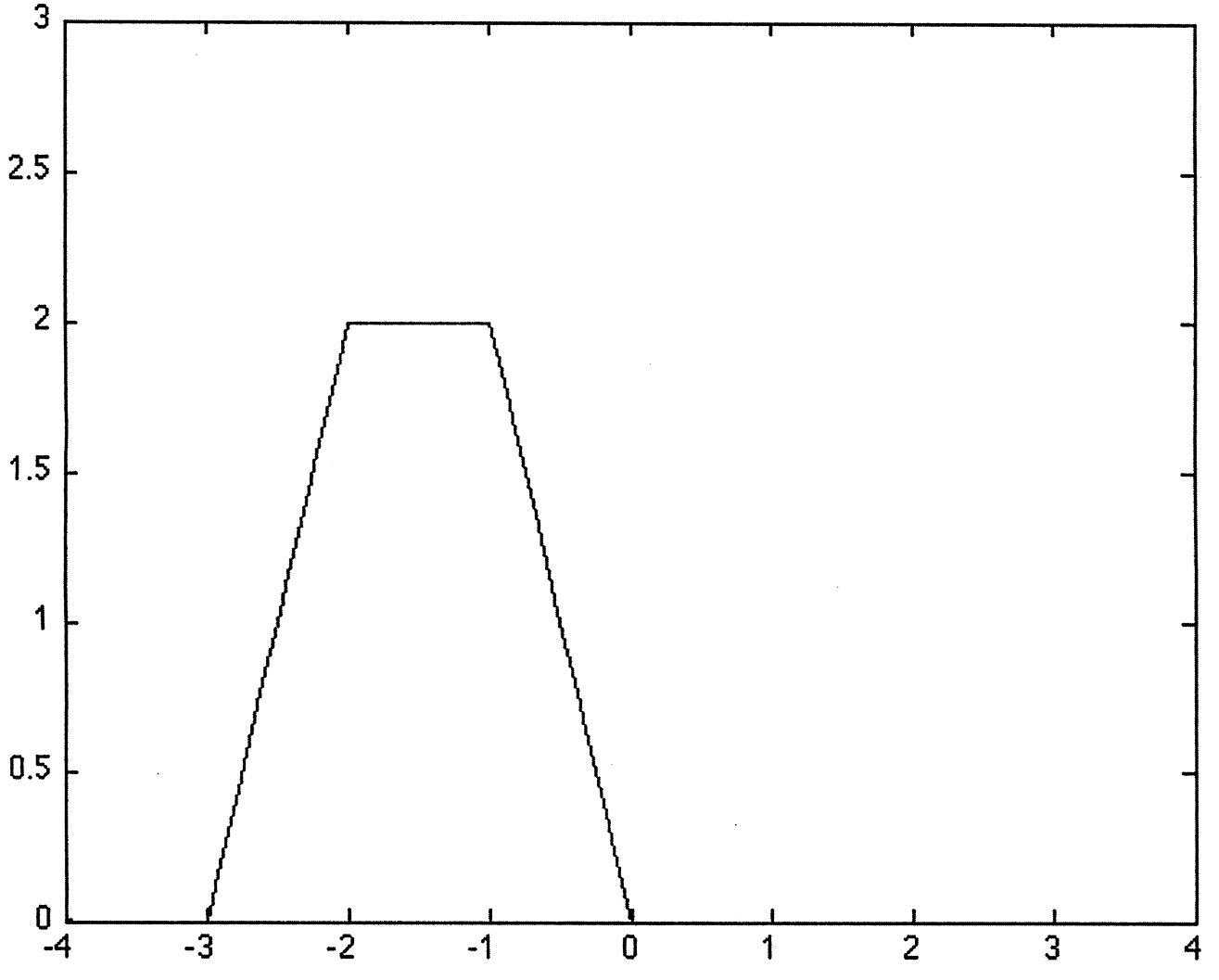
2. Consider the following function $g(t)$:

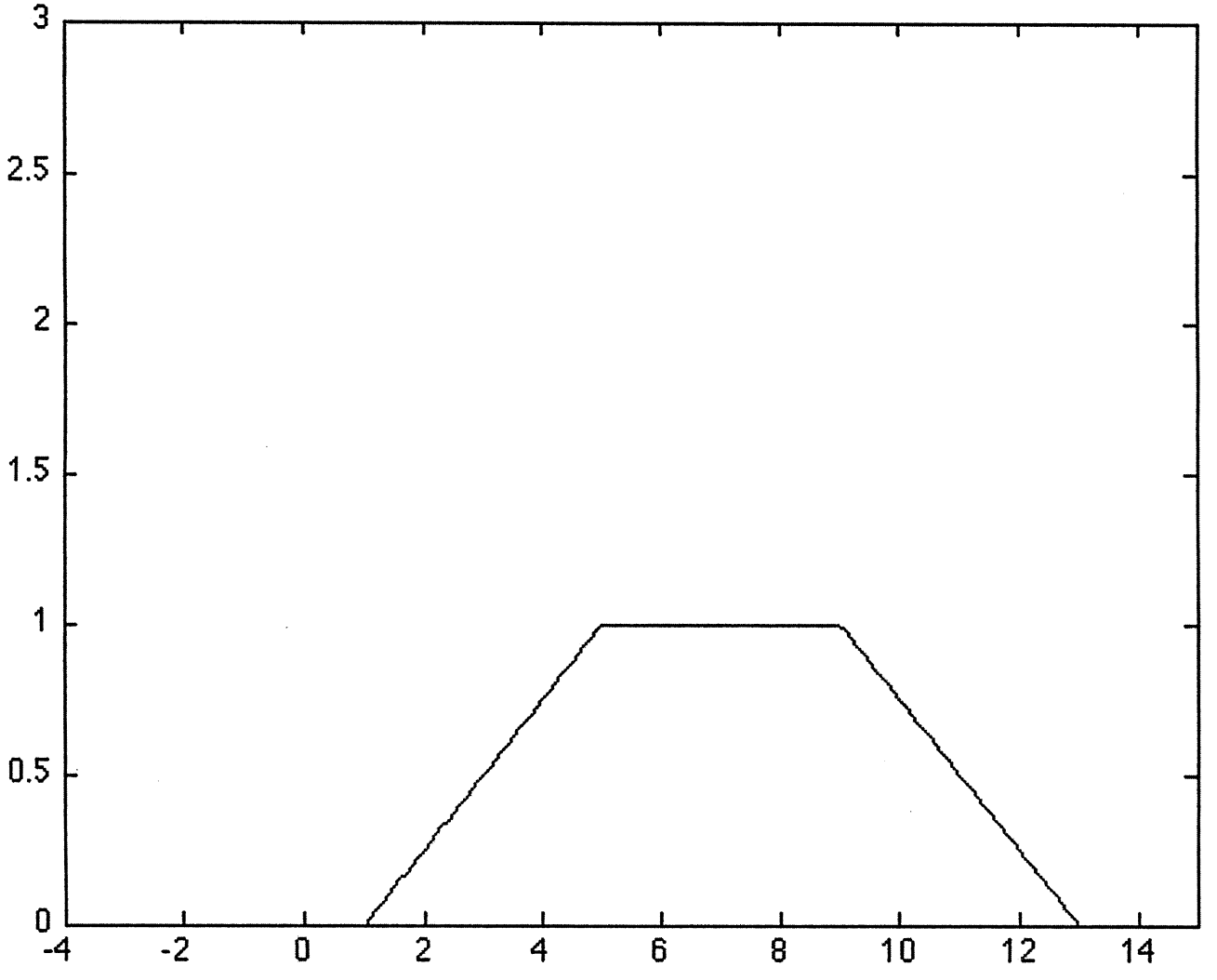



(a) (5 points) plot $f(t) = 2g(t+3)$ → shift left 3 time units and twice as big

(b) (5 points) plot $f(t) = g\left(\frac{t-1}{4}\right)$ ↓ shift right by 1, 4 times wider

(see next sheets)


3
2.5
2
1.5
1
0.5
0
-4
-3
-2
-1
0
1
2
3
4


3
2.5
2
1.5
1
0.5
0
-4
-2
0
2
4
6
8
10
12
14

3. Let $x(t)$ be band-limited to 200 Hz. Assume it is periodic but it neither an even or an odd function. From lecture we know

$$x(t) \xrightarrow{DFT} C_k$$

a) (5 points) Suppose $x(t)$ has a period of 2 seconds and it is sampled at a frequency of 1000 Hz. How many samples are taken over one period?
b) (5 points) What can you say about $|C_{200}|$? (NOTE: I'm not asking for its actual value.)
c) (5 points) What can you say about $|C_{330}|$?
d) (3 points) How could you increase the frequency resolution?

(a) 2000

(b) $\frac{f_s}{N} = \Delta f = 0.5\,Hz$

$\therefore C_{200}$ is harmonic @ 100 Hz

$|C_{200}| \geq 0$ because it is inside Bandwidth of signal

(c) $C_{330}$ is harmonic at 165 Hz still within bandwidth of signal so

$|C_{330}| \geq 0$

(d) deleted from exam (Do not grade)

4. A system's impulse response is $h(t) = \left(\cos^2 5t\right)u(t)$.

(a) (2 points) What is the system's output $y(t)$ for an arbitrary input signal $x(t)$?

(b) (5 points) What is the system's step response?

(c) (3 points) Is the system's step response an energy signal? (Justify your answer)

(a) $y(t) = h(t) * x(t)$

(b) $y(t) = \int_0^t \cos^2 5\tau \, d\tau$

$= \frac{1}{10}\left(5t + \sin 5t \cos 5t\right)\Big|_0^t$

$= \frac{1}{10}\left(5t + \sin 5t \cos 5t\right)$

(c) no. It's a periodic function so the area under the curve over $\pm\infty$ is infinite

$\therefore$ can't be an energy signal

5. A LTI system is governed by the following differential equation

$$\frac{d^4y}{dt^4} + 2\frac{dy}{dt} = 3x$$

(a) (5 points) Find the transfer function $H(s)$

(b) (5 points) Find the frequency response function $H(\omega)$.

(c) (2 points) Find the magnitude of the frequency response at $\omega = 10$ rad/s.

(a) $s^4 Y(s) + 2sY(s) = 3X(s)$

$(s^4 + 2s)Y(s) = 3X(s)$

$$\frac{Y(s)}{X(s)} = H(s) = \frac{3}{s^4 + 2s}$$

(b) $$H(\omega) = \frac{3}{(j\omega)^4 + j2\omega} = \frac{3}{\omega^4 + j2\omega}$$

(c) $$\frac{3}{10^4 + j20} = H(\omega)\Big|_{\omega=10}$$

$$|H(\omega)|\Big|_{\omega=10} = \frac{3}{\sqrt{(10^4)^2 + (20)^2}}$$

$$\approx \frac{3}{10^4}$$

# A Brief Table of Integrals

*An arbitrary constant may be added to each integral*

1. $\int x^n\,dx = \frac{1}{n+1}x^{n+1}$ (if $n \neq -1$)
2. $\int \frac{1}{x}\,dx = \ln|x|$
3. $\int u\,dv = uv - \int v\,du$
4. $\int e^x\,dx = e^x$
5. $\int \sin x\,dx = -\cos x$
6. $\int \cos x\,dx = \sin x$
7. $\int \tan x\,dx = -\ln|\cos x|$
8. $\int \sec x\,dx = \ln|\tan x + \sec x|$
9. $\int \sec^2 x\,dx = \tan x$
10. $\int \ln x = x\ln x - x$
11. $\int \frac{1}{a+bx}\,dx = \frac{1}{b}\ln|a+bx| \quad (b \neq 0)$
12. $\int \frac{1}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} \quad (a > 0)$
13. $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a} \quad (a > 0)$
14. $\int \frac{1}{a^2-x^2}\,dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right|$
15. $\int \frac{1}{x(a+bx)}\,dx = \frac{1}{a}\ln\left|\frac{x}{a+bx}\right|$
16. $\int x^n e^{ax}\,dx = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,dx$
17. $\int x^n \sin ax\,dx = -\frac{1}{a}x^n\cos ax - \frac{n}{a}\int x^{n-1}\cos ax\,dx$
18. $\int x^n \cos ax\,dx = \frac{1}{a}x^n\sin ax - \frac{n}{a}\int x^{n-1}\sin ax\,dx$
19. $\int \sin ax \sin bx\,dx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$
20. $\int \sin ax \cos bx\,dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$
21. $\int \cos ax \cos bx\,dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$
22. $\int \sin^2 ax\,dx = \frac{1}{2a}(ax - \sin ax\cos ax)$
23. $\int \cos^2 ax\,dx = \frac{1}{2a}(ax + \sin ax\cos ax)$
24. $\int e^{ax}\sin bx\,dx = \frac{e^{ax}(a\sin bx - b\cos bx)}{a^2+b^2}$
25. $\int e^{ax}\cos bx\,dx = \frac{e^{ax}(b\sin bx + a\cos bx)}{a^2+b^2}$
26. $\int x^n \ln ax\,dx = x^{n+1}\left(\frac{\ln ax}{n+1} - \frac{1}{(n+1)^2}\right)$