FIGURE 4.1 A closed-loop system.

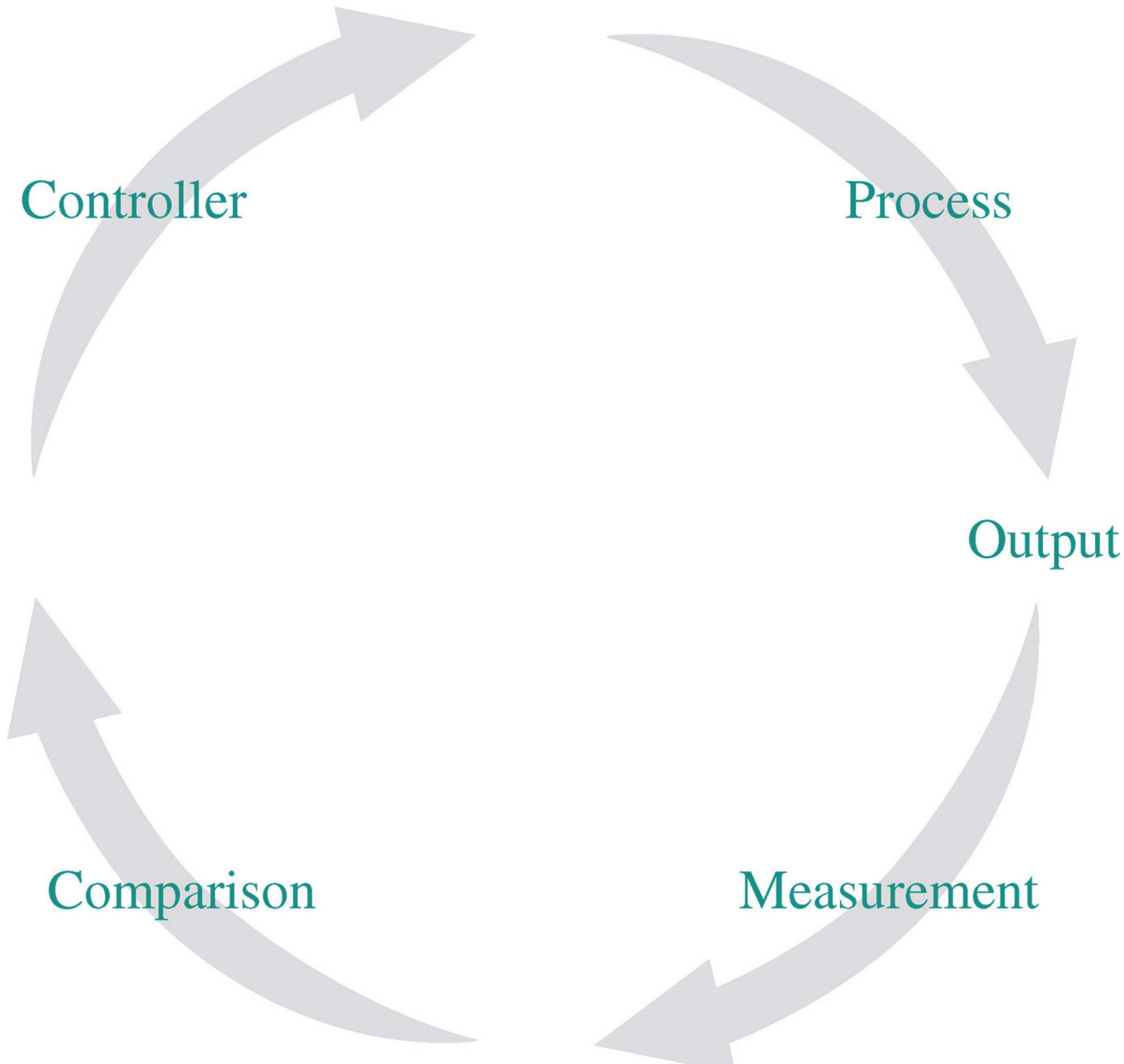


FIGURE 4.2 An open-loop system with a disturbance input, $T_d(s)$. (a) Signal-flow graph. (b) Block diagram.

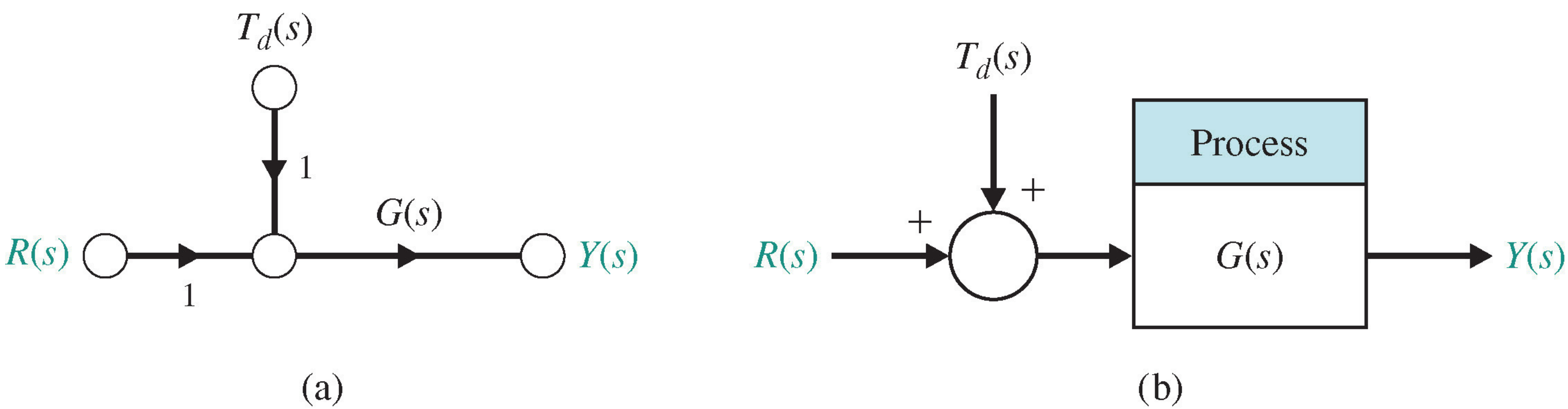


FIGURE 4.3 Open-loop control system (without feedback). (a) Signal-flow graph. (b) Block diagram.

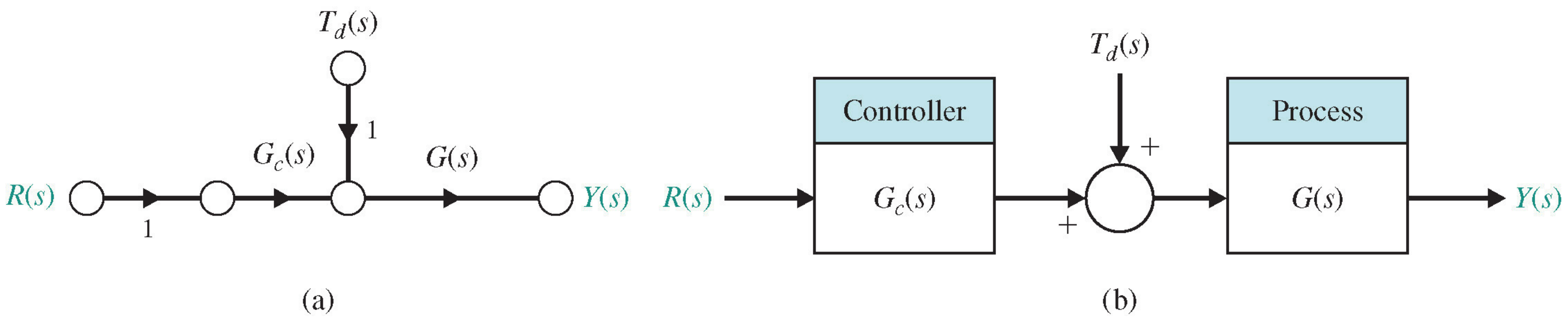


FIGURE 4.4 A closed-loop control system. (a) Signal-flow graph. (b) Block diagram.

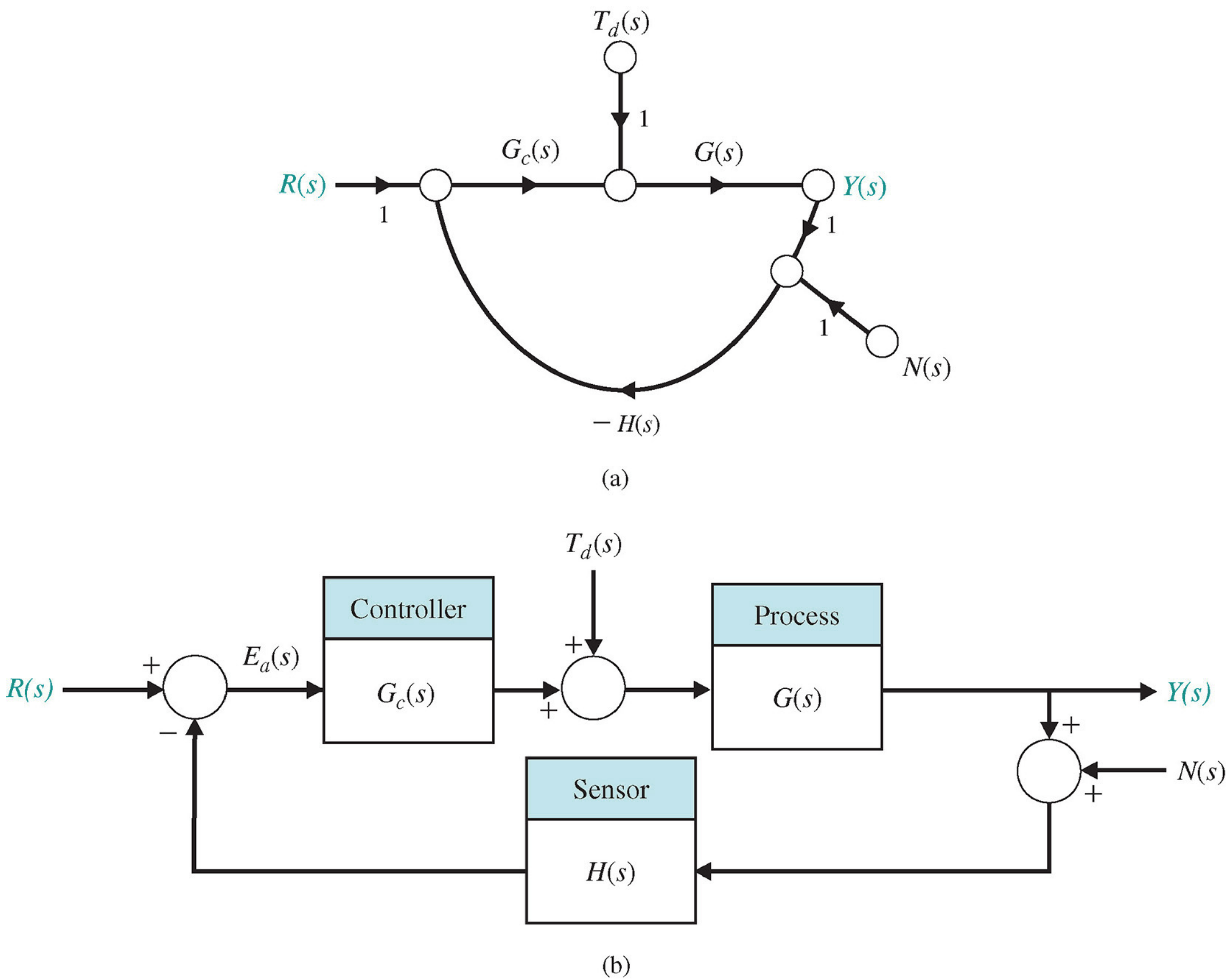
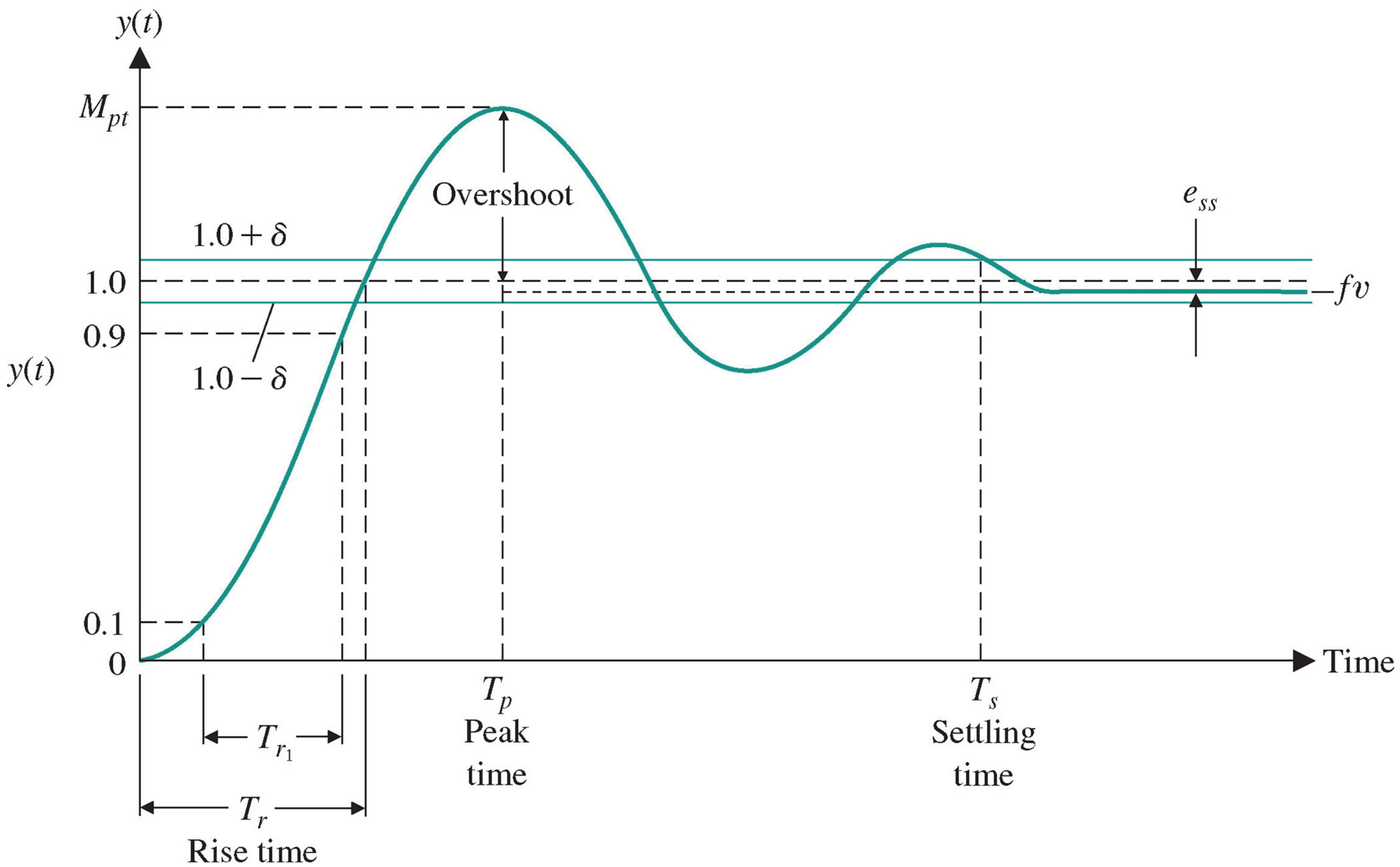
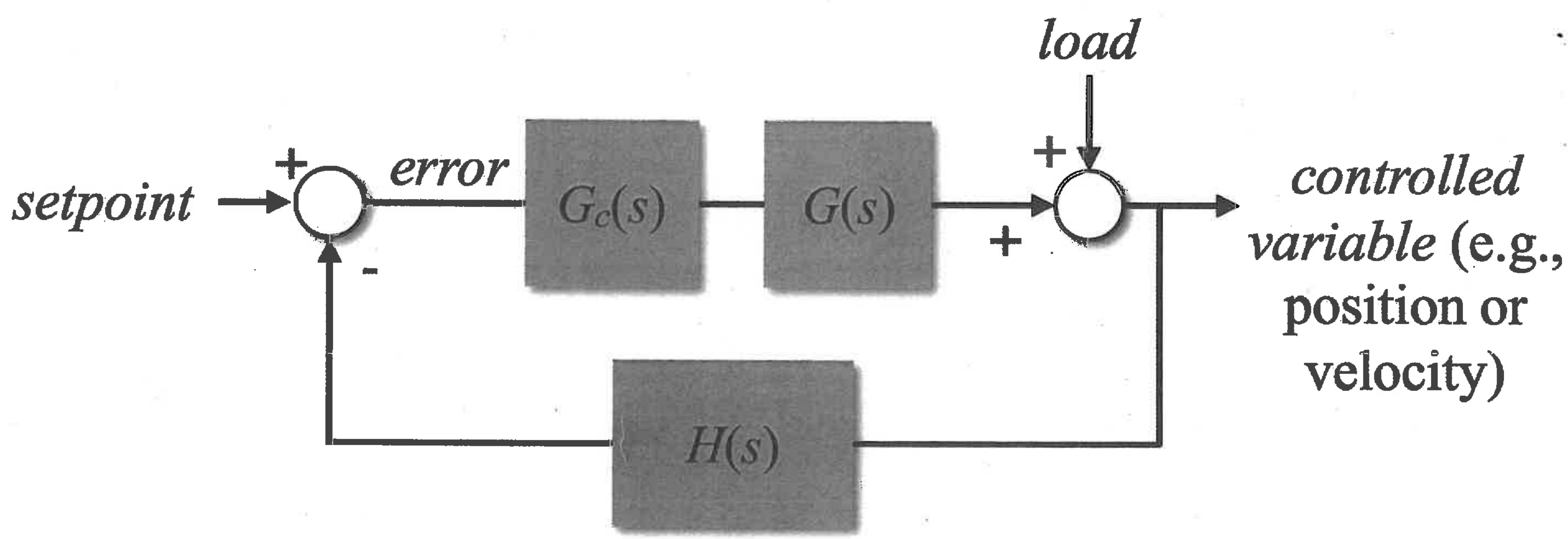


FIGURE 5.6 Step response of a second-order system.





In *regulator systems* the load varies and the setpoint is fixed.

In *tracking* or *servo systems* the load if fixed and the setpoint varies

Error constants

- Step-error (position-error) constant

$$K_p := \lim_{s \to 0} G(s)$$

- Ramp-error (velocity-error) constant

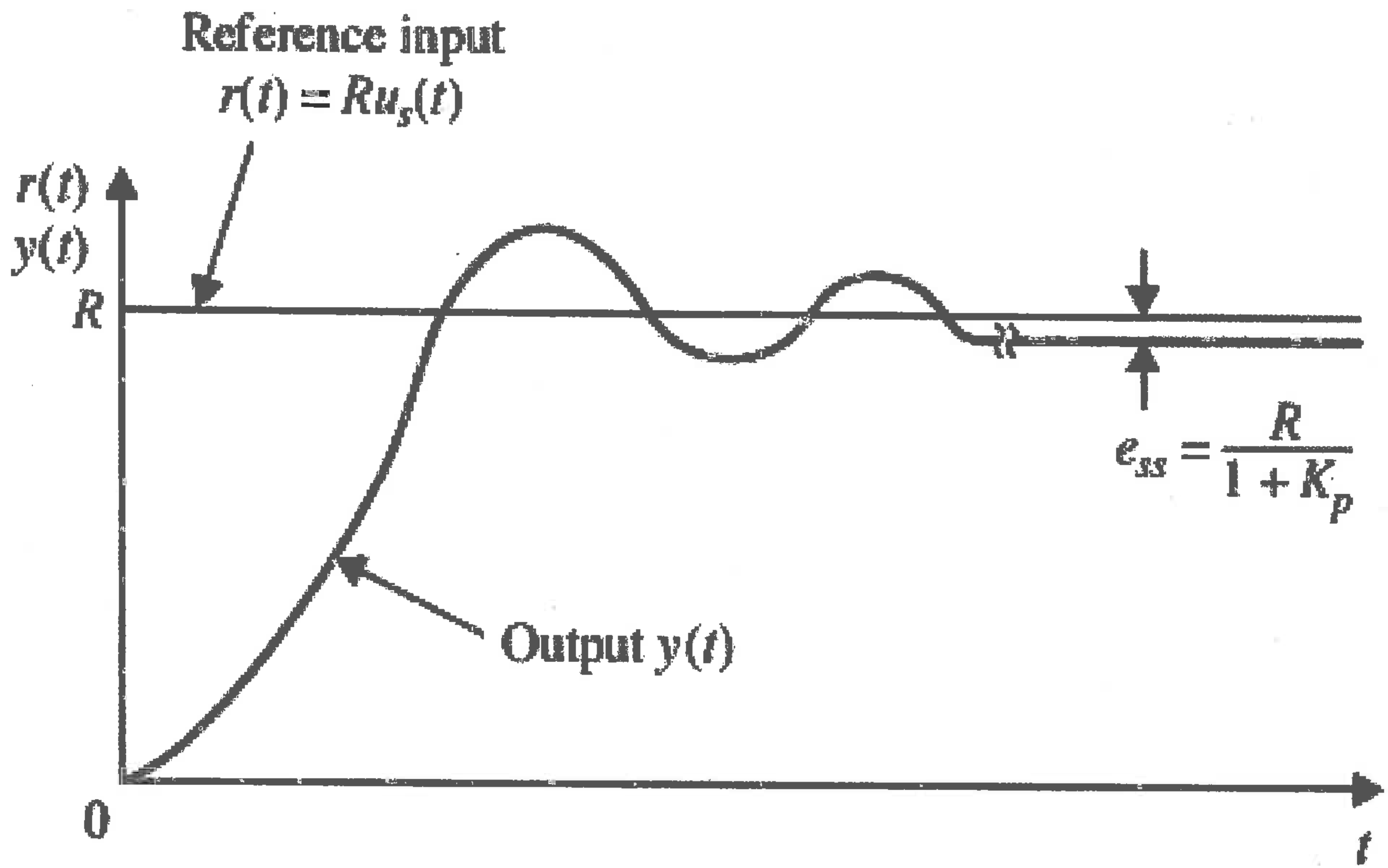
$$K_v := \lim_{s \to 0} sG(s)$$

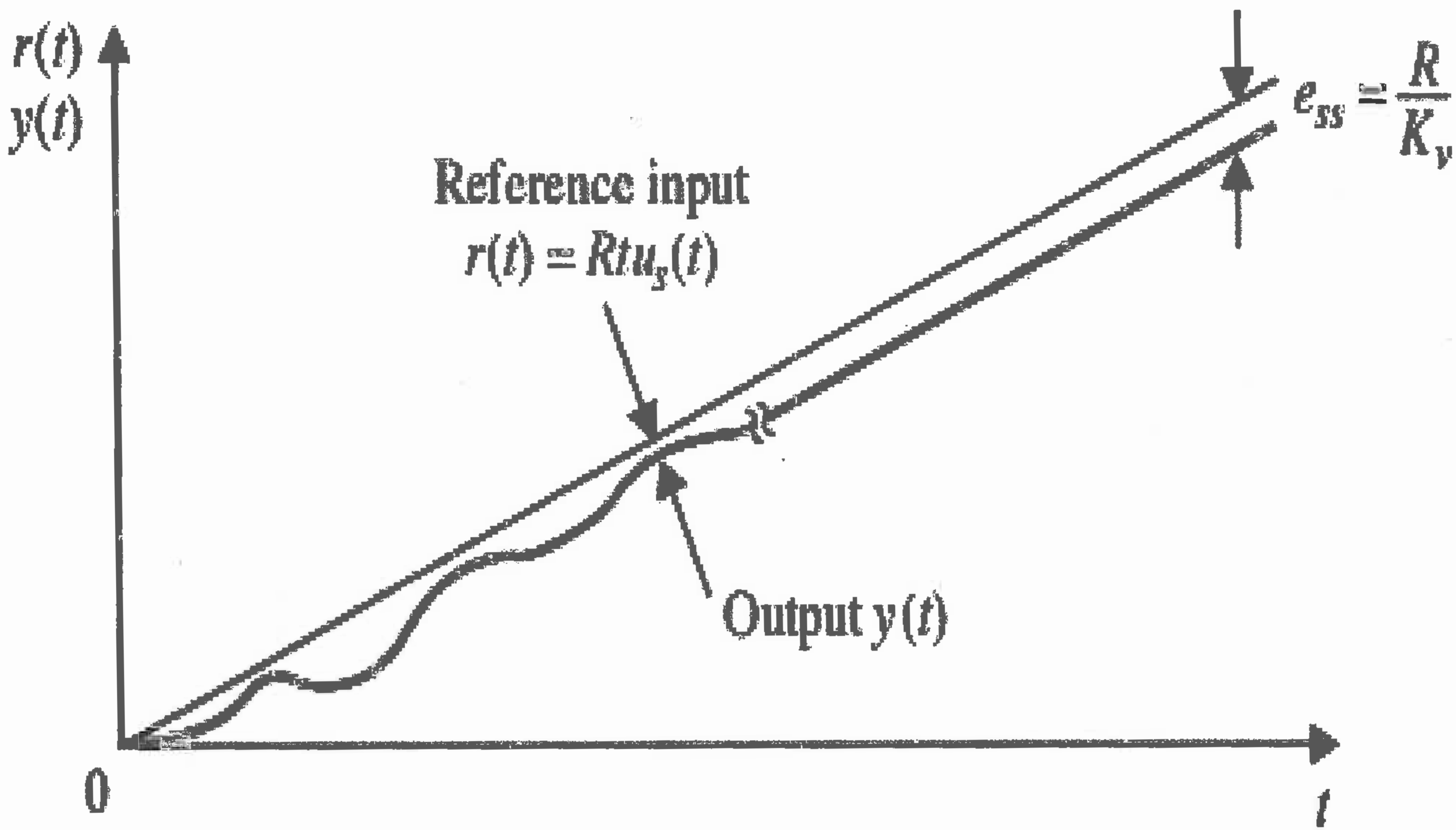
- Parabolic-error (acceleration-error) constant

$$K_a := \lim_{s \to 0} s^2 G(s)$$

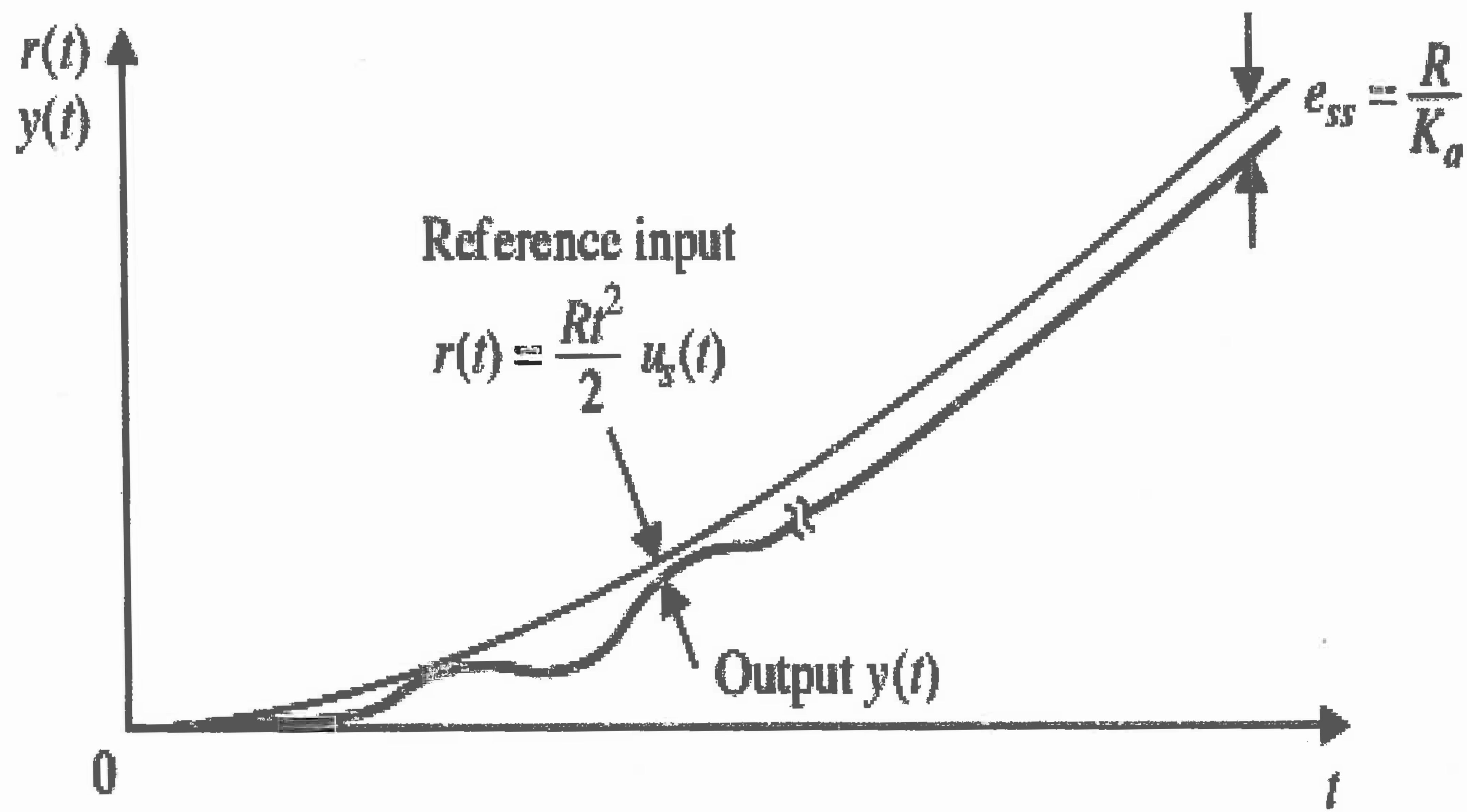
- Kp, Kv, Ka : *ability to reduce steady-state error*

indicates


Reference input
r(t) = Ru_s(t)
r(t)
y(t)
R
e_ss = R / (1 + K_p)
Output y(t)
0
t



$r(t)$
$y(t)$
Reference input
$r(t) = Rtu_s(t)$
$e_{ss} = \frac{R}{K_v}$
Output $y(t)$
0
t



$r(t)$
$y(t)$
$e_{ss} = \frac{R}{K_a}$
Reference input
$r(t) = \frac{Rt^2}{2} u_s(t)$
Output $y(t)$
0
t

Summary of Steady-State Errors

Type Number	Input: Step, $r(t) = 1$, $R(s) = 1/s$	Ramp, $r(t) = t$, $R(s) = 1/s^2$	Parabola, $r(t) = t^2/2$, $R(s) = 1/s^3$
0	$e_{ss} = \frac{1}{1 + K_p}$	∞	∞
1	$e_{ss} = 0$	$\frac{1}{K_v}$	∞
2	$e_{ss} = 0$	0	$\frac{1}{K_a}$

⇓	⇓	⇓
perfect tracking only for type 1 & 2 finite error for type 0	type 0 can't track finite error for type 1 perfect tracking for type 2	type 0 & 1 can't track type 2 can but with finite error

FIGURE 5.18 Block diagram of steering control system for a mobile robot.

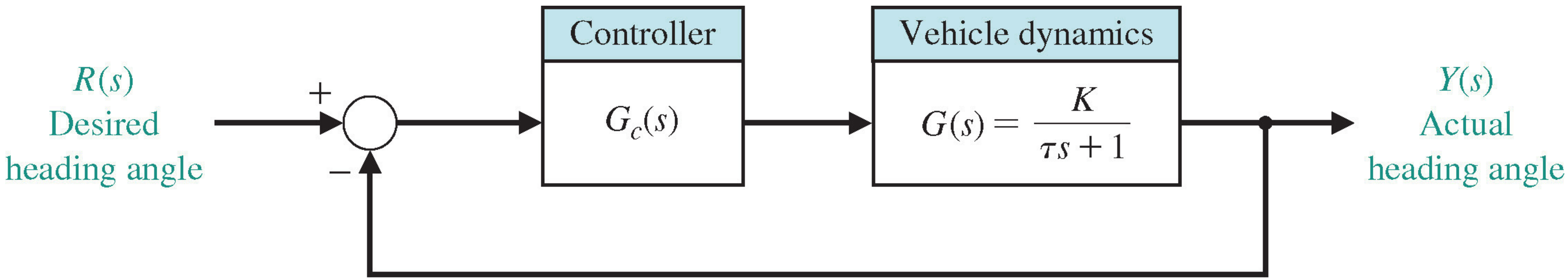
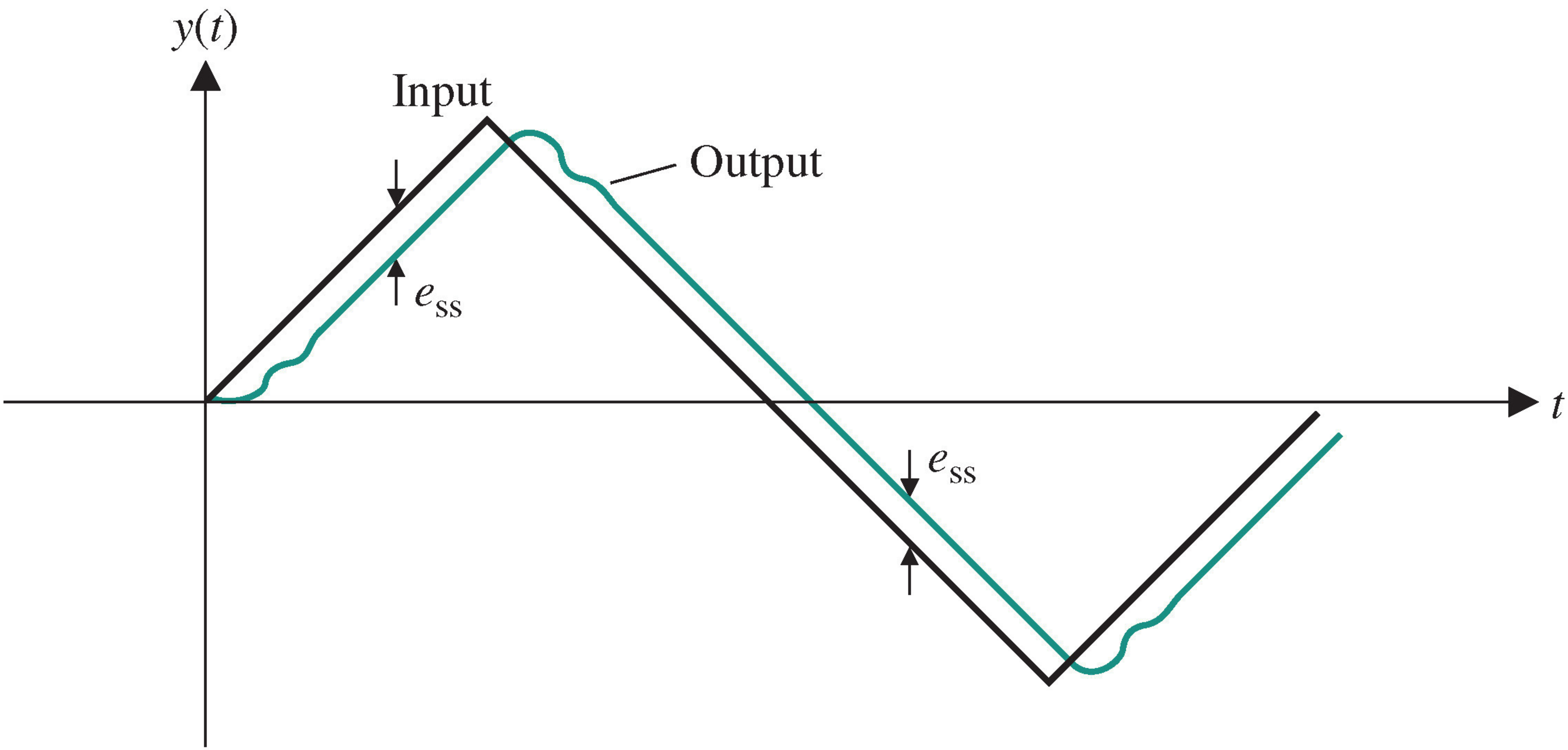
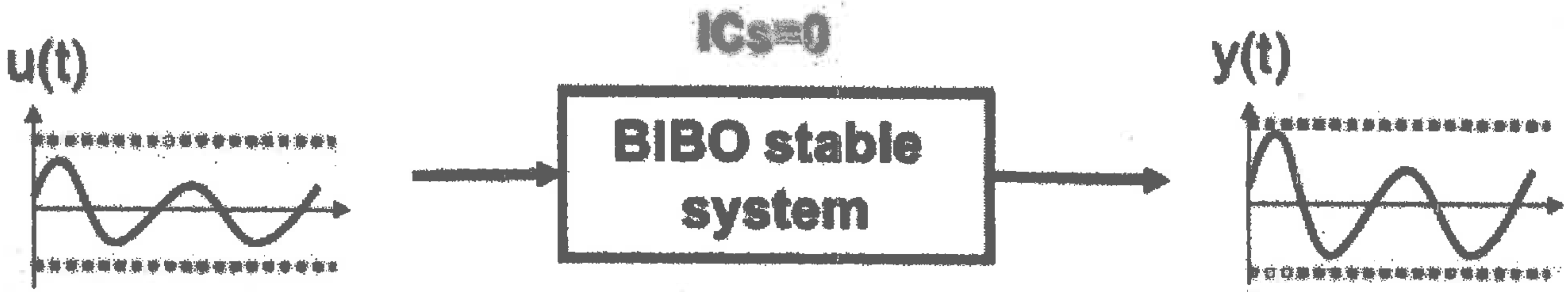


FIGURE 5.19 Triangular wave response.



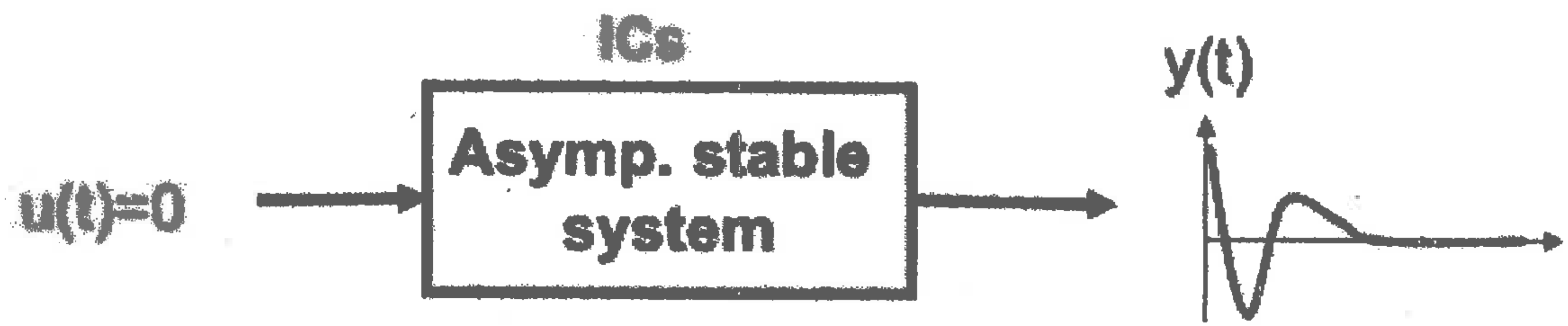
Mathematical definitions of stability

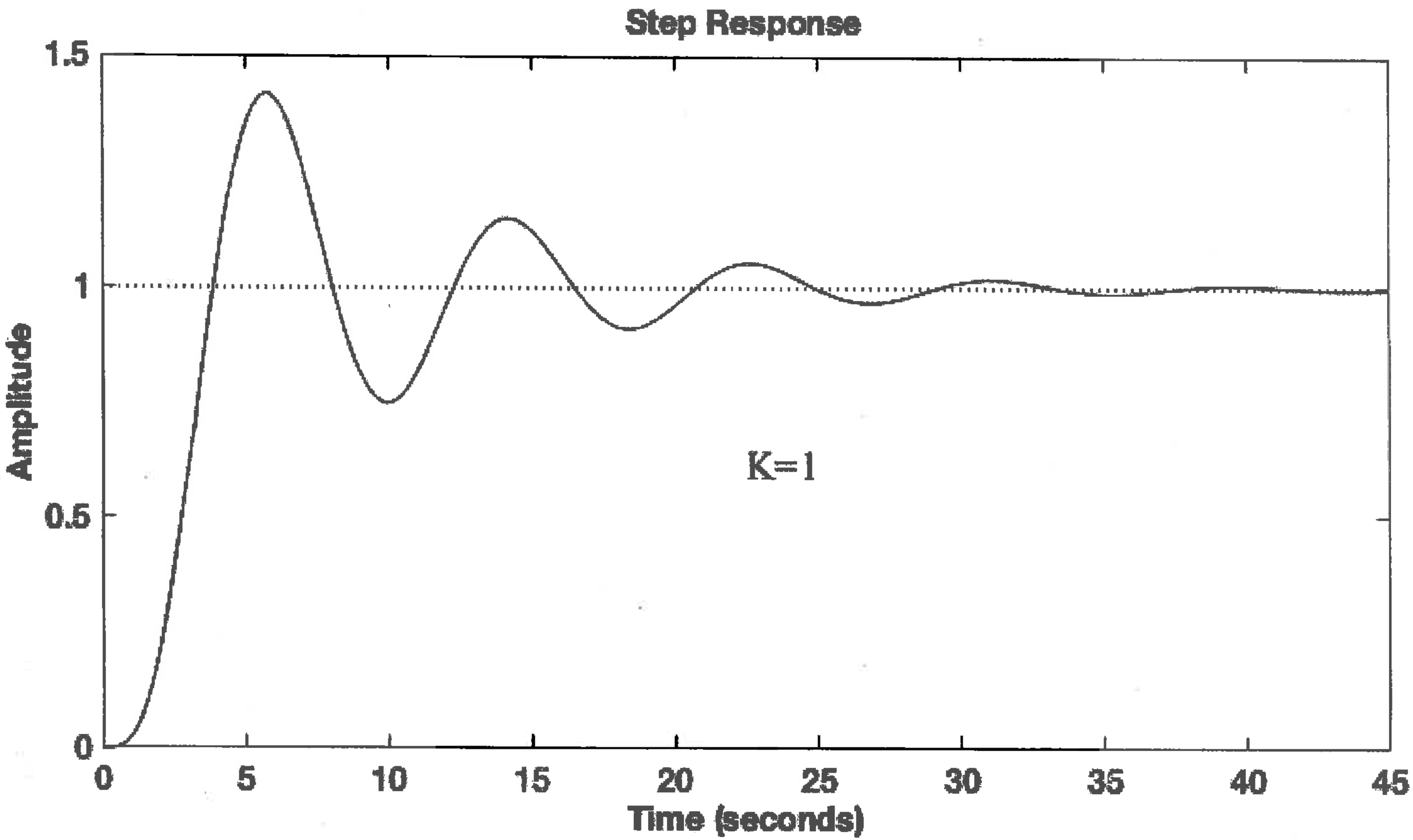
- **BIBO** (Bounded-Input-Bounded-Output) **stability** :
  Any bounded input generates a bounded output.

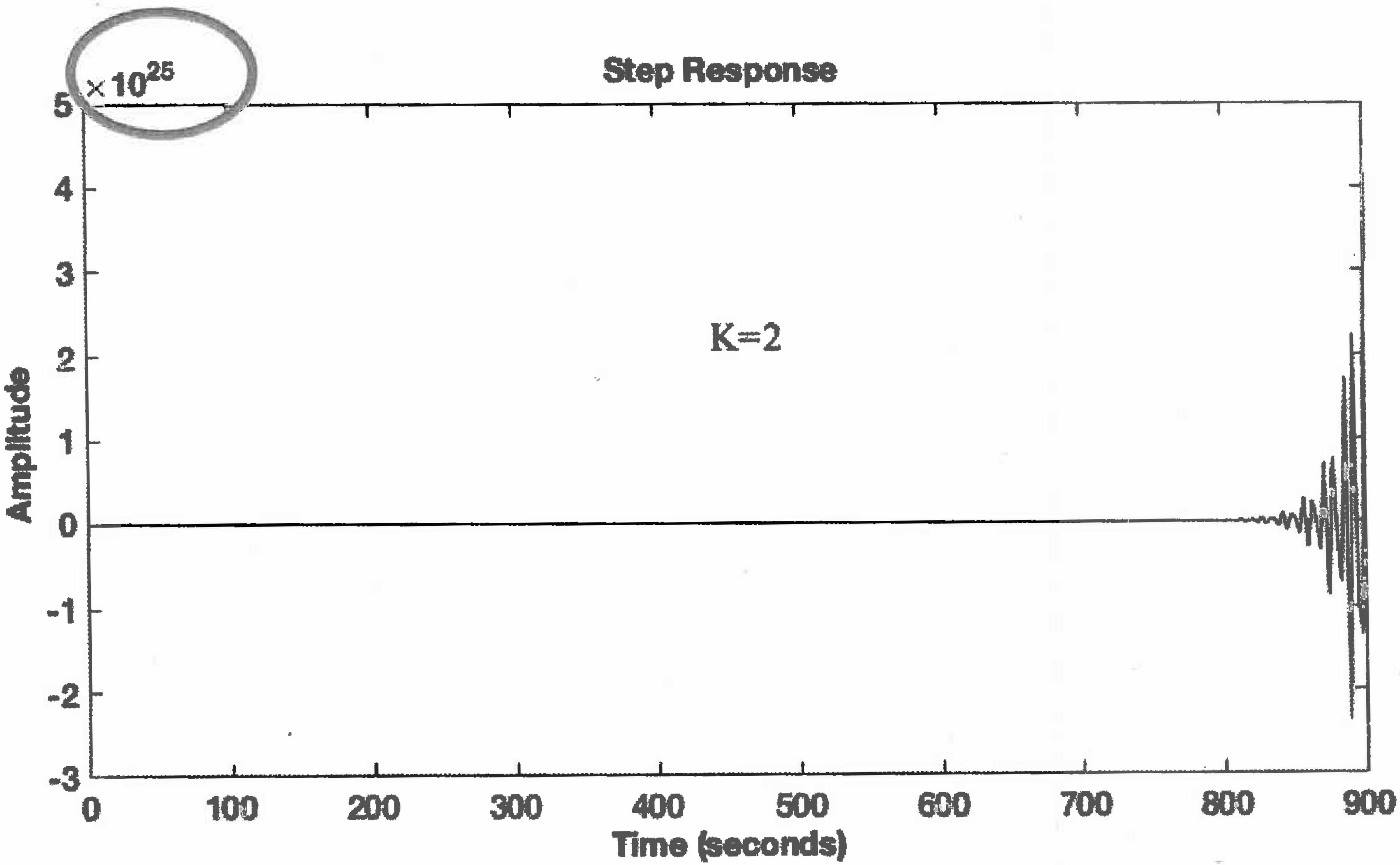


- **Asymptotic stability** :

  Any ICs generates y(t) converging to zero.




Step Response
1.5
1
0.5
0
Amplitude
K=1
0
5
10
15
20
25
30
35
40
45
Time (seconds)



Step Response
×10^25
5
4
3
2
1
0
-1
-2
-3
Amplitude
K=2
0
100
200
300
400
500
600
700
800
900
Time (seconds)

- ***Stability via epsilon method***
  - ***Problem:*** Determine the stability of the closed-loop transfer function.

$$T(s)=\frac{10}{s^5+2s^4+3s^3+6s^2+5s+3}$$

Stability via epsilon method

- ***Solution*** *(continued):*

s^5	1	3	5
s^4	2	6	3
s^3	~~0~~ ϵ	$\frac{7}{2}$	0
s^2	$\frac{6\epsilon-7}{\epsilon}$	3	0
s^1	$\frac{42\epsilon-49-6\epsilon^2}{12\epsilon-14}$	0	0
s^0	3	0	0

Table 6.1 The Routh–Hurwitz Stability Criterion

n	Characteristic Equation	Criterion
2	$s^2 + bs + 1 = 0$	$b > 0$
3	$s^3 + bs^2 + cs + 1 = 0$	$bc - 1 > 0$
4	$s^4 + bs^3 + cs^2 + ds + 1 = 0$	$bcd - d^2 - b^2 > 0$
5	$s^5 + bs^4 + cs^3 + ds^2 + es + 1 = 0$	$bcd + b - d^2 - b^2e > 0$
6	$s^6 + bs^5 + cs^4 + ds^3 + es^2 + fs + 1 = 0$	$(bcd + bf - d^2 - b^2e)e + b^2c - bd - bc^2f - f^2 + bfe + cdf > 0$

Note: The equations are normalized by $(\omega_n)^n$.